# Worker Turnover and Unemployment Insurance
# Online Appendix

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## 1 Comparative Statics regarding cdf F.

In this section, we conduct a comparative static exercise with respect to the distribution function. Consider two cumulative distribution functions, $F_1$ and $F_2$. We say that $F_2$ dominates $F_1$, $F_1 \lesssim F_2$, if the two following assumptions hold.

**Assumption 1** *$F_1$ is less than $F_2$ in the strict mean residual life order. That is,*

$$\int_{\pi \geq t} \pi \frac{dF_1(\pi)}{1 - F_1(t)} < \int_{\pi \geq t} \pi \frac{dF_2(\pi)}{1 - F_2(t)}, \ \forall t$$

**Assumption 2** *$F_2$ first-order stochastically dominates $F_1$. That is,*

$$F_1(t) \geq F_2(t), \ \forall t, \ \textit{and strict inequality for some } t$$

We refer to the first assumption as *strict* mean residual life order because it requires a strict inequality instead of weak inequality. This stochastic order implies that the truncated expected match-specific productivity is always larger with the distribution $F_2$. The second assumption establishes that more productive matches are more likely in an economy with cdf $F_2$.[1]

Lemma 1.1 states that if productivity values are drawn from the cdf $F_2$, then vacancy creation is larger, the reservation productivity and the match-termination rate are lower, and wages and job-finding rates are higher in equilibrium relative to the economy with $F_1$. As a result, the unemployment rate is lower.

**Lemma 1.1** *Comparative Statics. Let $(w_i, q_i, R_i, s_i)$ denote the equilibrium tuple of the economy indexed by $i$. If two economies differ in the cumulative distribution such that $F_1 \lesssim F_2$, then $w_1 < w_2$,*

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[1]It is worth noticing that, under some additional condition, the mean residual life order implies first order stochastic dominance. See Shaked and Shanthikumar (1994, Ch. 1.D) for further details. As an example, consider a uniform distribution and a perturbation of it, $F_1 \sim U[0, 1]$ and $F_2(\pi) = -\epsilon\pi^2/2 + (1 + \epsilon/2)\pi$. We have $F_1 \lesssim F_2$ for $\epsilon > 0$.

*$q_2 < q_1$, $R_2 < R_1$ (unless both are zero), and $s_1 < s_2$ in equilibrium. The job-finding probability is higher, and the match-termination and unemployment rates are lower under $F_2$. .*

**Proof** Consider two economies that differ in the signal distribution function, and $F_1 \lesssim F_2$. Let $(q_1, w_1, R_1, s_1)$ and $(q_2, w_2, R_2, s_2)$ be the respective equilibrium tuples. These tuples solve the program stated in Proposition 4.1 in the paper, which we rewrite here:[2]

$$\max_{s,q,w} \quad \left\{ -\phi(s) + s\nu(q)\Big(v(w) - v(z) + \beta \int_R \big(v(w) - v(z)\big) dF_i(\pi)\Big)\right\} \tag{1}$$

$$\text{s. to} \quad \eta(q)\Big(\mathbb{E}_i(\pi)y - w + \beta \int_R \big(\pi y - w\big) dF_i(\pi)\Big) \geq k, \text{ and the internal efficiency condition}$$

where we define $\mathbb{E}_i(\pi) \equiv \displaystyle\int \pi dF_i(\pi)$. Let us also rewrite here the equilibrium conditions:

$$\phi'(s) \;=\; \nu(q)\Big(v(w) - v(z) + \beta \int_R \big(v(w) - v(z)\big) dF_i(\pi)\Big) \tag{2}$$

$$\frac{k}{1+\beta(1-F_i(R))} \;=\; \eta(q)(1-\varphi(q))\Big(\frac{v(w)-v(z)}{v'(w)} + y\frac{\overline{\pi} + \beta(1-F_i(R))\mathbb{E}_i(\pi|R))}{1+\beta(1-F_i(R))} - w\Big) \tag{3}$$

Furthermore, if $R > 0$, we can rewrite the latter, using the internal efficiency condition, as

$$\frac{k}{1+\beta(1-F_i(R))} \;=\; \eta(q)(1-\varphi(q))\Big(\frac{\overline{\pi} + \beta(1-F_i(R))\mathbb{E}_i(\pi|R))}{1+\beta(1-F_i(R))} - R\Big)y \tag{4}$$

First, notice that the equilibrium tuple $(q_1, w_1, R_1, s_1)$ satisfies the two constraints of the program when $F = F_2$. This implies

$$\nu(q_2)\big(v(w_2) - v(z)\big)\big(1 + \beta(1 - F_2(R_2))\big) \geq \nu(q_1)\big(v(w_1) - v(z)\big)\big(1 + \beta(1 - F_2(R_1))\big) \tag{5}$$

We only study the case in which both thresholds are strictly positive $R_1, R_2 > 0$, since the proofs for the other cases are small variations of this. First, we prove by contradiction that $q_2 < q_1$. Suppose instead that $q_1 \leq q_2$.

The equilibrium condition (4) implies that

$$\begin{aligned}
(1+\beta(1-F_1(R_1)))\Big(\frac{\overline{\pi} + \beta(1-F_1(R_1))\mathbb{E}_1(\pi|R_1))}{1+\beta(1-F_1(R_1))} - R_1\Big) \quad &\geq \\
(1+\beta(1-F_2(R_2)))\Big(\frac{\overline{\pi} + \beta(1-F_2(R_2))\mathbb{E}_2(\pi|R_2))}{1+\beta(1-F_2(R_2))} - R_2\Big) \quad &> \\
(1+\beta(1-F_1(R_2)))\Big(\frac{\overline{\pi} + \beta(1-F_1(R_2))\mathbb{E}_1(\pi|R_2))}{1+\beta(1-F_1(R_2))} - R_2\Big), &
\end{aligned}$$

where the last inequality results from $F_1 \lesssim F_2$. The second parenthesis in either line is a decreasing

[2] We have imposed the equilibrium result of time-invariant wages for simplicity.

function in $R$ and, hence, we obtain $R_1 < R_2$ from comparing the first and the last expressions. This together with the internal efficiency condition implies $w_2 < w_1$. Then, it follows from the inequality (5) that $R_1 > R_2$, which is a contradiction. Therefore, $q_2 < q_1$.

We also proceed by contradiction to prove that $R_2 < R_1$. Suppose the opposite holds, $R_1 \leq R_2$. Then, again, the internal efficiency condition implies $w_2 < w_1$. However, we can rewrite the equilibrium equation (3), using the internal efficiency and the free entry conditions, as

$$w = Ry + \frac{\varphi(q)}{1-\varphi(q)} \frac{k}{\eta(q)} \frac{1}{1+\beta(1-F(R))}$$

Notice that the right hand side is increasing in $R$ and decreasing in $q$. Therefore, since $q_2 < q_1$ and $R_1 \leq R_2$, it follows

$$\begin{aligned} w_2 &= R_2 y + \frac{\varphi(q_2)}{1-\varphi(q_2)} \frac{k}{\eta(q_2)} \frac{1}{1+\beta(1-F(R_2))} > \\ &\quad R_1 y + \frac{\varphi(q_1)}{1-\varphi(q_1)} \frac{k}{\eta(q_1)} \frac{1}{1+\beta(1-F(R_1))} = w_1, \end{aligned}$$

which is a contradiction. Therefore, $R_2 < R_1$. Finally, the internal efficiency condition implies $w_1 < w_2$, and $s_2 > s_1$ follows from the equilibrium equation (2).‖

## 2 Alternative Setting

In this section, we examine an alternative setting in which match quality is both an inspection good and an experience good. We first describe the differences with respect to the two-period economy analyzed in the paper, referred to here as the benchmark, and then characterize the planner's solution. Finally, we determine the equilibrium allocation and analyze its welfare properties. All proofs are in the Appendix section.

### 2.1 Environment

This is a frictional economy in which firms and workers search for a partner at the beginning of the first period. Meetings take place according to the same aggregate meeting technology as the one described in the benchmark model. The decision of forming a match depends on the match quality.

We build upon Jovanovic (1979) and Pries and Rogerson (2005), and model match quality as both an inspection good and an experience good. It can be either good, in which case a firm-worker pair produces $y$ units of output, or bad, and then output is 0. Match quality is unobservable at the meeting time. However, the parties draw a signal $\pi$, which conveys the probability of the match quality being good, from a differentiable cdf $F$ with support the unit interval. Match formation depends on the observed signal. If a match is not formed, the worker stays unemployed for the two periods and produces $z$ units of output at home. Otherwise, the expected productivity in the first period amounts to $\pi y$, and match quality is learned at the beginning of the second period. If it turns out to be bad, the match is terminated and the worker produces $z$ at home. Otherwise, the output produced by the match is $y$.

**Assumption.** To guarantee vacancy creation both in the equilibrium and the constrained efficient allocations, we assume that the expected discounted output produced at the market net of the vacancy-creation costs exceeds home productivity,

$$\mathbb{E}(\pi)y(1+\beta) - z(1+\beta\mathbb{E}(\pi)) > k \tag{6}$$

## 2.2 The Planner's Allocation

In this section, we analyze the social planner's allocation. The social planner's problem is very similar to the one in the benchmark economy. The additional constraint it faces is the following: it cannot learn the quality of a potential match prior to period two. It observes a signal for a match quality, and decides whether to form a match or not by setting a reservation probability $R$.

Search effort is unobservable to the planner. Workers are promised consumption $c_u$ if unemployed in either period, and $c_t(\pi)$ if employed in period $t$ with signal $\pi$. Therefore, the following incentive compatibility condition must hold.

$$\phi'(s) = \nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi) \tag{7}$$

Moreover, the following resource constraint must hold.

$$\begin{aligned}\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big)(z - c_u)\\ + s\nu(q)\int_R \Big(\pi y - c_1(\pi) + \beta\pi\big(y - c_2(\pi)\big)\Big)dF(\pi) \geq k\frac{s}{q}\end{aligned} \tag{8}$$

where, to save on notation, we denote the conditional expected probability as

$$\mathbb{E}(\pi|R) \equiv \int_R \pi \frac{dF(\pi)}{1-F(R)}$$

Therefore, the planner's allocation is a solution to the following program:

$$\max_{\substack{s,q,R,c_u\\ c_1(\cdot),c_2(\cdot)}} \Big\{ -\phi(s) + s\nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi)\Big\} + v(c_u)(1+\beta)$$
$$\text{s. t. conditions (7), (8)} \tag{9}$$

The following proposition establishes existence of the planner's solution, and characterizes it.

The results do not qualitatively differ from the ones in the benchmark.

**Proposition 2.1** *There exists a constrained efficient allocation* $(s^p, q^p, R^p, c_u^p, c_1^p(\cdot), c_2^p(\cdot))$*. The consumption levels are such that* $z < c_u^p < c^p = c_1^p(\pi) = c_2^p(\pi)$ *for all* $\pi$*. The reservation probability* $R^p$ *satisfies*

$$\frac{v(c^p) - v(c_u^p)}{v'(c^p)} + c_u^p - z + y\frac{R(1+\beta)}{1+\beta R} - c^p \geq 0, \text{ and } R \geq 0, \text{ with complementary slackness} \quad (10)$$

*Furthermore,* $y\frac{R^p(1+\beta)}{1+\beta R^p} < z$*, and*

$$\eta(q^p)(1 - F(R^p))(1 - \varphi(q^p))\left(\mathbb{E}(\pi|R^p) - (1 + \beta\mathbb{E}(\pi|R^p))\frac{R^p}{1+\beta R^p}\right)y(1+\beta) = k, \text{ if } R^p > 0 \quad (11)$$

## 2.3 Market Economy

In this section, we determine the equilibrium allocation. As in the benchmark, we focus on internally efficient equilibria. We first describe some additional environment details.

### 2.3.1 Environment

For expositional ease, firms compete for workers by committing to a single wage $w$ to be paid in each period conditional on being employed. Let $W \equiv [z, y]$ denote the space of contractual offers.

**Match Formation.** Upon meeting, a worker and a firm draw a signal $\pi$ from the cdf $F$. A match is formed if and only if the joint value of the worker-firm pair is positive. Conditional on wages, the joint value of the pair as a function of signal $\pi$ amounts to

$$\mathcal{S}_w(\pi) = \pi y - w + \frac{v(w) - v(z)}{v'(w)} + \beta\pi\left(y - w + \frac{v(w) - v(z)}{v'(w)}\right) \quad (12)$$

The joint value is the sum of firm's profits and worker's utility if employed net of her utility if unemployed adjusted by the marginal utility of wages. Since $\mathcal{S}_w$ is an increasing function of $\pi$, the optimal decision regarding match formation has the reservation value property: a match is formed if and only if the signal $\pi$ is above a threshold $R_w$, where

$$R_w = \begin{cases} 0 & \text{, if } \mathcal{S}_w(0) > 0 \\ 1 & \text{, if } \mathcal{S}_w(1) < 0 \\ \pi & \text{, such that } \mathcal{S}_w(\pi) = 0, \text{ o.w.} \end{cases} \quad (13)$$

We will refer to this match-formation condition as the *internal efficiency* condition. It can be shown that there is a negative relationship between $R$ and $w$,

$$\frac{dR_w}{dw} = \frac{\frac{v''(w)(v(w)-v(z))}{v'(w)^2}(1+\beta R)}{y+\beta(y-w+\frac{v(w)-v(z)}{v'(w)})} < 0$$

**Value Functions.** A profit-maximizing firm posts a vacancy in submarket $w$ at cost $k$ if it expects to obtain non-negative profits. A vacancy is filled with probability $\eta(q(w))(1-F(R_w))$. The firm's value is

$$V = -k + \max_w \eta(q(w)) \int_{R_w} \Big(\pi y - w + \beta\pi\big(y-w\big)\Big) dF(\pi) \tag{14}$$

Free entry implies that the expected returns from posting a vacancy must be zero, $V = 0$.

Workers become employed with probability $s\nu(q(w))(1-F(R_w))$, and remain employed in the second period with probability $\mathbb{E}(\pi|R_w)$. Job-seekers search in submarket $w$ if the expected discounted value promised there coincides with their market value $U$. That is, the following condition must hold for any submarket $w \in W$.

$$\begin{aligned} U \quad \geq \quad & \max_s \Big\{ -\phi(s) + s\nu(q(w)) \int_{R_w} \Big(v(w) - v(z) + \beta\pi\big(v(w)-v(z)\big)\Big) dF(\pi) \Big\} + v(z)(1+\beta), \\ & \text{and } q(w) \geq 0, \text{ with complementary slackness} \end{aligned} \tag{15}$$

### 2.3.2 Equilibrium

Next, we define the equilibrium concept.

**Definition 1** *An internally efficient competitive search equilibrium consists of a market value $U$, a search intensity function $s : W \to [0,1]$, a queue length function $Q : W \to \mathbb{R}_+ \bigcup \{\infty\}$, a contract $w \in W$, and a reservation value policy such that*

*i) Match-formation decision:*

*For any $w' \in W$, the associated reservation probability $R_{w'}$ is determined by condition (13).*

*ii) Firm's profit maximization and zero-profit condition:*

*For any $w' \in W$,*

$$\eta(Q(w')) \int_{R_{w'}} \Big(\pi y - w' + \beta\pi\big(y - w'\big)\Big) dF(\pi) \leq k,$$

*which holds with equality at $w$.*

*iii) Worker's optimal search:*

*For any $w'$, $Q(w')$ makes condition (15) hold. In particular,*

$$U = \max_s \Big\{ -\phi(s) + s\nu(Q(w)) \int_{R_w} \Big( v(w) - v(z) + \beta\pi\big(v(w) - v(z)\big)\Big) dF(\pi) \Big\} + v(z)(1+\beta)$$

The definition of equilibrium follows closely the one in the benchmark economy; hence, all clarifications are omitted. The following proposition characterizes the equilibrium allocation.

**Proposition 2.2** *An internally efficient competitive search equilibrium solves the following program, and, conversely, a solution of the program takes part of an equilibrium allocation.*

$$\max_{s,q,w} \quad \Big\{ -\phi(s) + s\nu(q) \int_R \Big( v(w) - v(z) + \beta\pi\big(v(w) - v(z)\big)\Big) dF(\pi) \Big\} \tag{16}$$
$$\text{s. to} \quad \eta(q) \int_R \Big( \pi y - w + \beta\pi\big(y - w\big)\Big) dF(\pi) \geq k, \text{ and condition (13)}$$

*There exists an internally efficient equilibrium. It is determined by the zero-profit condition, the internal efficiency condition (13), and*

$$\phi'(s) = \nu(q) \int_R \Big( v(w) - v(z) + \beta\pi\big(v(w) - v(z)\big)\Big) dF(\pi) \tag{17}$$
$$k = \eta(q)(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Big( \frac{v(w)-v(z)}{v'(w)} + y\frac{\mathbb{E}(\pi|R)(1+\beta)}{1+\beta\mathbb{E}(\pi|R)} - w \Big) \tag{18}$$

*If $R \in (0,1)$, condition (13) becomes*

$$\frac{v(w)-v(z)}{v'(w)}(1+\beta R) + Ry(1+\beta) - w(1+\beta R) = 0 \tag{19}$$

*Furthermore, in equilibrium, $Ry(1+\beta) < z(1+\beta R) < w(1+\beta R)$ and $R < 1$.*

Next, we perform two comparative static exercises. We obtain that either the more risk averse workers are or the smaller their home production is, the larger the number of jobs created in the economy is and the lower the wages and the threshold are. The result on the effects on the reservation probability is not obvious as firms trade off providing insurance in the first period against insurance in the second period. These results imply that the more risk averse workers are, the more important the insurance motive against present risks becomes. Similarly, a lower outside option makes workers even less risky in their search strategies, and the period-one-insurance role of the equilibrium contracts becomes more important. Our interpretation is as follows. In an economy with not fully insured present (period one) and future (period two) unemployment risks, risk-averse jobless workers factor such risks in their search decisions. Workers prefer to apply to lower-wage jobs inasmuch as they are easier to get despite their higher termination rates the stronger the insurance motive is. That is, matches are of lower quality in expected terms, which leads to poorer post-unemployment outcomes: lower wages and output per worker, and a higher job-separation rate in the second period, $1 - \mathbb{E}(\pi|R)$. The overall effects on total output in period one are uncertain because, on one hand, output per worker is lower, and, on the other hand, the unemployment rate is also lower.

**Lemma 2.3** *Comparative Statics. Let $(w_i, q_i, R_i, s_i)$ denote the equilibrium tuple of the economy indexed by $i \in \{A, B\}$. If two economies differ in either*

1. *the workers' utility function $v$, and $v_B$ is a concave monotonic transformation of $v_A$,*

2. *the home production value, and $z_B < z_A$,*

*then $w_B < w_A$, $q_B < q_A$, and $R_B < R_A$ (and $R_B = R_A$ if and only if both are 0).*

### 2.3.3 Implementation

We next examine the role of the government to achieve constrained efficiency in equilibrium. The results are the same as in the benchmark economy. Thus, we just state them without further discussions.

**Proposition 2.4** *The internally efficient competitive search equilibrium is not constrained efficient. Furthermore, if $R^p > 0$, then*

*i) the equilibrium threshold $R$ and queue length $q$ are lower than their constrained efficient counterparts. Thus, the job-finding rate and job-termination rate, and, hence, worker turnover, are inefficiently high.*

*ii) constrained efficiency is attained if a government sets an unemployment insurance system funded through lump sum and negative income taxes, whereas layoff taxes must be zero.*

## References


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# 3 Appendix

## 3.1 Proofs.

**Proof of Proposition 2.1.**

To show that there exists a constrained efficient allocation, let us assume that the consumption functions while employed are indeed constant in $\pi$, $c_t(\pi) = c_t$. This is a necessary condition for an interior solution of the planner's problem (9) as shown below, and assuming it now greatly simplifies the proof of existence. The objective function is continuous in all its arguments. Its domain is a non-empty set because of assumption (6). That is, the tuple $(q, R, c_u, c_1, c_2, s)$ such that all consumption levels equal $z$, $s = 0$, and the reservation probability takes any value, satisfies the two constraints. Furthermore, the domain is a compact set. Therefore, the Weierstrass Theorem applies to ensure the existence of a solution of the maximization problem.

We now characterize the planner's solution. The Lagrangian is

$$\begin{aligned}\mathcal{L} = \quad & -\phi(s) + s\nu(q)\int_R \Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi) + v(c_u)(1+\beta) \\ & +\xi_1\bigg(\phi'(s) - \nu(q)\int_R\Big(v(c_1(\pi)) - v(c_u) + \beta\pi\big(v(c_2(\pi)) - v(c_u)\big)\Big)dF(\pi)\bigg) \\ & +\xi_2\bigg((z-c_u)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big) + \\ & +s\nu(q)\int_R\Big(\pi y - c_1(\pi) + \beta\pi\big(y - c_2(\pi)\big)\Big)dF(\pi) - k\frac{s}{q}\bigg)\end{aligned}$$

where $\xi_1$ and $\xi_2$ are the multipliers. Differentiating the Lagrangian with respect to the variables, we obtain a number of necessary conditions for the constrained efficient allocation. First, the functional derivatives with respect to $c_1(\pi)$ and $c_2(\pi)$, we obtain

$$\begin{aligned}c_1(\pi) = c_2(\pi) = c \\ v'(c)(s-\xi_1) = s\xi_2\end{aligned} \tag{20}$$

Second, the derivative of the Lagrangian with respect to $s$ delivers the following first order condition.

$$\begin{aligned}\xi_1\phi''(s) \quad &= \quad \xi_2\bigg(k/q + \nu(q)(1-F(R))\Big[(z - c_u + c)(1+\beta\mathbb{E}(\pi|R)) - \mathbb{E}(\pi|R)y(1+\beta)\Big]\bigg) \\ \Leftrightarrow \xi_1 s\phi''(s) \quad &= \quad \xi_2(z-c_u),\end{aligned} \tag{21}$$

where the last condition comes out after using the second constraint, which must hold with equality. This implies that $\xi_1 < 0$ if and only if $c_u > z$.

Now, differentiating with respect to $c_u$, using (20), and simplifying, we have

$$\big(v'(c) - v'(c_u)\big)\nu(q)(1-F(R))(1+\beta\mathbb{E}(\pi|R))(s-\xi_1) + (1+\beta)(v'(c_u) - \xi_2) = 0 \tag{22}$$

The left hand side is $\frac{\partial\mathcal{L}}{\partial c_u}$. To show that $z < c_u$, it suffices to show that the derivative of the Lagrangain evaluated at $c_u \leq z$ is strictly positive. To see this, notice that expression (21) together with $c_u \leq z$ implies $\xi_1 \geq 0$, and it also follows that $v'(c) \geq \xi_2$ from equation (20). The inequality $v'(c_u) > v'(c) \geq \xi_2$ follows from the concavity of the utility function and the assumption that $z < c$. Therefore, the derivative evaluated at $c_u \leq z$ is strictly positive.

Similarly, to show that $c_u < c$, it suffices to show that the derivate of the Lagrangian for $c_u \geq c$ is strictly negative. This result follows from $\xi_2 > v'(c) > v'(c_u)$ due to expressions (20) and (21).

Next, we look at the necessary condition for the reservation probability. After some manipulations, we obtain

$$\frac{\partial \mathcal{L}}{\partial R} = -\nu(q)(1+\beta R)(s-\xi_1)dF(R)\Big(v(c)-v(c_u)+v'(c)\Big(c_u - z + y\frac{R(1+\beta)}{1+\beta R} - c\Big)\Big) \leq 0,$$
$$\text{and } R \geq 0, \text{ with complementary slackness} \tag{23}$$

The derivative is non-positive if and only if $\frac{v(c)-v(c_u)}{v'(c)} + c_u - z + y\frac{R(1+\beta)}{1+\beta R} - c \geq 0$, which leads to expression (10). Let $R$ be such that $y\frac{R(1+\beta)}{1+\beta R} \geq z$. Notice that $\frac{v(c)-v(c_u)}{v'(c)} + c_u - c > 0$ because of the concavity of the utility function. Therefore, the derivative is strictly negative. That is, such value cannot be the maximizing $R$ because the Lagrangian increases by reducing the reservation value. Therefore, if $R > 0$, then $y\frac{R(1+\beta)}{1+\beta R} < z$. Notice that this inequality also holds if $R = 0$.

The last necessary FOC condition is with respect to the ratio $q$. After some manipulations, we obtain

$$(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Big(\frac{v(c)-v(c_u)}{v'(c)} - z + c_u + \frac{\mathbb{E}(\pi|R)(1+\beta)}{1+\beta\mathbb{E}(\pi|R)}y - c\Big) = \frac{k}{\eta(q)} \tag{24}$$

Indeed, if $R > 0$, then using condition (23), we can rewrite it as

$$(1-\varphi(q))(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Big(\frac{\mathbb{E}(\pi|R)(1+\beta)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R(1+\beta)}{1+\beta R}\Big)y = \frac{k}{\eta(q)}.\| \tag{25}$$

**Proof of Proposition 2.2.** For the first part of the proof and for the sake of simplicity, we are to impose the necessary condition that the wage variables are constant functions of $\pi$. Such first order conditions are derived below. Moreover, as in the benchmark, we will show later below that the internal efficiency condition is redundant as the first order condition with respect to variable $R$ in the maximization problem without this condition is this same equilibrium condition. Therefore, we will proceed with the maximization problem without constraint (13).

Let $(Q, R, w_1, w_2, s, U)$ be an equilibrium allocation. We first show that the tuple $(q, R, w_1, w_2, s)$ solves the maximization problem (16), where $q = Q((w_1, w_2), R)$. The proof is by contradiction. Suppose that there exists another tuple $(q', R', w_1', w_2', s')$ such that the constraint holds, and, without loss of generality,

$$\begin{aligned}
&\nu(q')\int_{R'}\Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big)dF(\pi) \\
&> \nu(q)\int_{R}\Big(v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big)dF(\pi) \\
&\geq \nu(Q(x'))\int_{R'}\Big(v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big)dF(\pi),
\end{aligned}$$

where $Q(x')$ denotes the off-the-equilibrium queue length for contract $x' = (w_1', w_2'), R')$. The last inequality results from the definition of equilibrium. It follows from the monotonicity of function $\nu$ that $q' < Q(x')$. The first equilibrium condition in Definition 1 ensures that

$$\begin{aligned}
k = \quad &\eta(q)\int_{R}\Big(\pi y(1+\beta) - w_1 - \beta\pi w_2\Big)dF(\pi) \\
&\geq \eta(Q(x'))\int_{R'}\Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big)dF(\pi) \\
&> \eta(q')\int_{R'}\Big(\pi y(1+\beta) - w_1' - \beta\pi w_2'\Big)dF(\pi) \geq k,
\end{aligned}$$

where the last inequality comes from the monotonicity of function $\eta$ and $q' < Q(x')$. This is a contradiction.

Therefore, the equilibrium tuple solves program (16).

Now, let $(q, R, w_1(\cdot), w_2(\cdot), s)$ be a solution of the maximization problem (16). We define the equilibrium worker's market value as

$$U = -\phi(s) + s\nu(q) \int_R \Big( v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big) dF(\pi) + v(z)(1+\beta)$$

Likewise, we define the value of the queue length function $Q$ at any submarket $x'$ as the value $q'$ that satisfies

$$U = \max_s -\phi(s) + s\nu(q') \int_{R'} \Big( v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi) + v(z)(1+\beta),$$

if it exists, and $q' = 0$, otherwise.

It remains to show that firms maximize profits. That is, there is no submarket $x'$ such that

$$\eta(Q(x')) \int_{R'} \Big( \pi y(1+\beta) - w_1' - \beta\pi w_2' \Big) dF(\pi) > k$$

Suppose there is such a $x'$. Then, by the limit conditions and the continuity of function $\eta$, there must exist $q' < Q(x')$ such that

$$\eta(q') \int_{R'} \Big( \pi y(1+\beta) - w_1' - \beta\pi w_2' \Big) dF(\pi) = k$$

Since $(q', R', w_1'(\cdot), w_2'(\cdot), s')$ satisfies the constraint of the maximization problem, it follows that

$$\begin{aligned}
&\nu(q) \int_R \Big( v(w_1) - v(z) + \beta\pi\big(v(w_2) - v(z)\big)\Big) dF(\pi) \\
&\geq \nu(q') \int_{R'} \Big( v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi) \\
&> \nu(Q(x')) \int_{R'} \Big( v(w_1') - v(z) + \beta\pi\big(v(w_2') - v(z)\big)\Big) dF(\pi),
\end{aligned}$$

which contradicts the definition of $Q(x')$.

Existence of equilibrium follows from the Weierstrass Theorem because the objective function is continuous and the domain is a non-empty compact set because of assumption (6).

We now characterize the equilibrium and allow for wages to be contingent on $\pi$. The Lagrangian of the maximization problem is

$$\begin{aligned}
\mathcal{L} = \quad & -\phi(s) + s\nu(q) \int_R \Big( v(w_1(\pi)) - v(z) + \beta\pi\big(v(w_2(\pi)) - v(z)\big)\Big) dF(\pi) \\
& + \xi\Big( \eta(q) \int_R \Big( \pi y(1+\beta) - w_1(\pi) - \beta\pi w_2(\pi) \Big) dF(\pi) - k \Big),
\end{aligned}$$

where $\xi$ is the Lagrange multiplier. We differentiate the Lagrangian to obtain the necessary conditions. First, from the functional derivatives with respect to $w_1(\pi)$ and $w_2(\pi)$, we obtain

$$\begin{aligned}
w_1(\pi) = w_2(\pi) = w \\
v'(w) = \xi q
\end{aligned} \tag{26}$$

Next, differentiating the Lagrangian with respect to $R$, we obtain, after some manipulations,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial R} \quad = \quad & -\nu(q)(1+\beta R) dF(R) \Big( v(w) - v(z) + v'(w) \Big( y \frac{R(1+\beta)}{1+\beta R} - w \Big) \Big) \leq 0, \\
& \text{and } R \geq 0, \text{ with complementary slackness}
\end{aligned} \tag{27}$$

Notice that this necessary condition is the internal efficiency condition (13). The necessary first order condition with respect to $q$ is, after some manipulations,

$$\eta(q)\frac{1-\varphi(q)}{\varphi(q)}(1-F(R))(1+\beta\mathbb{E}(\pi|R))\frac{v(w)-v(z)}{v'(w)}=k \tag{28}$$

Likewise, the necessary first order condition related to $s$ is

$$\phi'(s)=\nu(q)\int_R\big(v(w)-v(z)\big)\big(1+\beta\pi\big)dF(\pi) \tag{29}$$

Therefore, the necessary first order conditions (27)-(29) plus the constraint (or zero-profit condition) characterize the equilibrium tuple $(q,w,R,s)$.

Finally, if $R>0$, it is convenient to combine the last two equilibrium conditions and the zero-profit condition to obtain

$$w\big(1+\beta\mathbb{E}(\pi|R)\big)=\varphi(q)\mathbb{E}(\pi|R)y(1+\beta)+(1-\varphi(q))y\frac{R(1+\beta)}{1+\beta R}\big(1+\beta\mathbb{E}(\pi|R)\big) \tag{30}$$

$$\eta(q)(1-\varphi(q))\frac{\mathbb{E}(\pi|R)-R}{1+\beta R}(1-F(R))y(1+\beta)=k \tag{31}$$

It is worth noticing that expression $\frac{\mathbb{E}(\pi|R)-R}{1+\beta R}(1-F(R))$ is a decreasing function of $R$.‖

**Proof of Lemma 2.3**

1. Consider two economies such that the utility function in the second economy, $v_B$, is a concave monotonic transformation of its counterpart in the first economy, $v_A$. That is, $v_B=g\circ v_A$, for some increasing and concave function $g$. Let $(q_A,R_A,w_A,s_A)$ and $(q_B,R_B,w_B,s_B)$ be the respective equilibrium vectors. Now, consider the maximization problem (16) without the internal efficiency condition (13). As the constraint does not depend on how risk averse workers are, the vector $(q_B,R_B,w_B,s_B)$ belongs to the domain of the program corresponding to the economy with utility function $v_A$, and vice versa. Therefore,

$$\nu(q_B)\big(v_B(w_B)-v_B(z)\big)\int_{R_B}\big(1+\beta\pi\big)dF(\pi)\geq\nu(q_A)\big(v_B(w_A)-v_B(z)\big)\int_{R_A}\big(1+\beta\pi\big)dF(\pi) \tag{32}$$
$$\nu(q_A)\big(v_A(w_A)-v_A(z)\big)\int_{R_A}\big(1+\beta\pi\big)dF(\pi)\geq\nu(q_B)\big(v_A(w_B)-v_A(z)\big)\int_{R_B}\big(1+\beta\pi\big)dF(\pi)$$

   The proof of $w_B<w_A$ is identical to the proof of Proposition 2 in Acemoglu and Shimer (1999); hence, we omit it.

   Now, we will show that $q_B<q_A$ and $R_B<R_A$. Let us distinguish between two cases.

   Case 1: $R_A,R_B>0$. We prove by contradiction that $q_B<q_A$. That is, suppose that the opposite is true: $q_B\geq q_A$. Then, the equilibrium condition (31) implies that $R_B\geq R_A$. Since $q_B\geq q_A$, $R_B\geq R_A$ and $w_B<w_A$, expression (32) cannot hold, which is a contradiction. Therefore, $q_B<q_A$, and $R_B<R_A$ follows again from condition (31).

   Case 2: $R_A=0<R_B$. We will show that this case cannot occur in equilibrium. Following the same argument as before, we obtain that $q_B<q_A$. The necessary equilibrium condition (28) implies

$$\frac{v_B(w_B)-v_B(z)}{v_B'(w_B)}>\frac{v_A(w_A)-v_A(z)}{v_A'(w_A)}$$

   Using this inequality and condition (27), we obtain

$$-y\frac{R_A(1+\beta)}{1+\beta R_A}+w_A\leq y\frac{R_B(1+\beta)}{1+\beta R_B}+w_B,$$

and as $w_B < w_A$, it follows that $R_B < R_A$, which is a contradiction.

2. Let $z_B < z_A$, and $(q_A, R_A, w_A, s_A)$ and $(q_B, R_B, w_B, s_B)$ be the respective equilibrium tuples. Consider the maximization problem (16) without the internal efficiency condition (13) for simplicity. As the constraint does not depend on the value of $z$, the equilibrium tuple $(q_A, w_A, R_A, s_A)$ satisfies the constraint of the program when $z = z_B$, and so does the vector $(q_B, w_B, R_B, s_B)$ when $z = z_A$. This implies

$$\nu(q_B)\big(v(w_B) - v(z_B)\big)\int_{R_B}(1+\beta\pi)dF(\pi) \geq \nu(q_A)\big(v(w_A) - v(z_B)\big)\int_{R_A}(1+\beta\pi)dF(\pi) \quad (33)$$

$$\nu(q_A)\big(v(w_A) - v(z_A)\big)\int_{R_A}(1+\beta\pi)dF(\pi) \geq \nu(q_B)\big(v(w_B) - v(z_A)\big)\int_{R_B}(1+\beta\pi)dF(\pi) \quad (34)$$

Multiplying these two inequalities and manipulating the outcome, we obtain

$$\big(v(w_B) - v(z_B)\big)\big(v(w_A) - v(z_A)\big) \geq \big(v(w_A) - v(z_B)\big)\big(v(w_B) - v(z_A)\big) \quad (35)$$
$$\Leftrightarrow \big(v(z_A) - v(z_B)\big)\big(v(w_A) - v(w_B)\big) \geq 0$$

As the utility function is increasing and $z_B < z_A$, it follows that $w_B \leq w_A$. Now, we show that $q_B \leq q_A$ by contradiction. Suppose that $q_B > q_A$. Then, the monotonicity of function $\nu$ and inequality (33) imply that $R_A > R_B$. We distinguish two cases.

Case 1: $R_B > 0$. The equilibrium condition (31) and $q_B > q_A$ imply that $R_A < R_B$ because its left hand side is a decreasing function in the ratio $R$. This is a contradiction. Therefore, $q_B \leq q_A$, and $R_B \leq R_A$ results again from condition (31).

Case 2: $R_B = 0$. The steps are analogous, but with a small difference. First, notice that the equilibrium condition (27) implies

$$v(w_B) - v(z_B) - v'(w_B)w_B \geq 0$$

Now, combining this inequality with equation (28) and the zero-profit condition, we obtain

$$\eta(q_B)(1-\varphi(q_B))\mathbb{E}(\pi|0)y(1+\beta) \leq k = \eta(q_A)(1-\varphi(q_A))\frac{\mathbb{E}(\pi|R_A) - R_A}{1+\beta R_A}(1-F(R_A))y(1+\beta),$$

where the last equality is expression (31). Because $q_B > q_A$, it must be the case that $R_A < R_B$. We reached a contradiction. Therefore, $q_B \leq q_A$. Obviously, in this case, $R_B = 0 \leq R_A$.‖

**Proof of Proposition 2.4**

Let $(q^p, R^p, c_u^p, c^p, s^p)$ denote the planner's solution to problem (9). We showed in the proof of Proposition 2.1 that $z < c_u^p$. Since unemployed workers just consume their home production in the equilibrium allocation, the planner's solution cannot be decentralized in the laissez-faire economy.

Consider now the case $R^p > 0$.

i) We are to show that the equilibrium threshold $R$ and queue length $q$ are lower than $R^p$ and $q^p$, respectively. Suppose that $R > R^p$ instead. Then, because the necessary efficiency and equilibrium conditions (11) and (18) establish a positive relationship between the cutoff and the queue length, it follows that $q > q^p$. Using the necessary efficiency and equilibrium conditions (10) and (19), we obtain

$$\frac{v(w) - v(z)}{v'(w)} + z - w = z - \frac{Ry(1+\beta)}{1+\beta R} < z - \frac{R^p y(1+\beta)}{1+\beta R^p} = \frac{v(c) - v(c^u)}{v'(c)} + c^u - c$$

Compare the left and right sides of this inequality. Since $z < c^u$ and $v$ is a concave function, it must be the case that $w < c$.

Now we show that such a tuple $(q, R, w)$ cannot take part of an equilibrium allocation. To see this, notice that $(q^p, R^p, c^p)$ satisfies the constraints of problem (16) as the intertemporal resource constraint together with $z < c^u$ implies

$$\eta(q^p)\int_{R^p}\Big(\pi y - c^p + \beta\pi(y - c^p)\Big)dF(\pi) > k$$

It also yields a higher value because $\big(1 - F(R^p)\big)(1 + \beta\mathbb{E}(\pi|R^p))(v(c^p) - v(z)) > \big(1 - F(R)\big)(1 + \beta\mathbb{E}(\pi|R))(v(w) - v(z))$. Therefore, we reach a contradiction, and $R \leq R^p$, which implies $q \leq q^p$ according to the first order conditions (11) and (18).

ii) Next, we consider a market economy in which a government implements a lump sum tax $T$, an income tax $\tau$ to the employed workers, a subsidy $b$ to the unemployed, and a layoff tax $L$ in order to decentralize the constrained efficient allocation $(q^p, R^p, c_u^p, c^p, s^p)$. This requires that the following conditions hold in equilibrium:

$$z + b - T = c_u^p \tag{36}$$

$$w(1-\tau) - T = c^p \tag{37}$$

$$\begin{aligned}(T-b)\Big(\big(1 - s\nu(q)(1-F(R))\big)(1+\beta) + s\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R))\Big)&\\ +Ls\nu(q)(1-F(R))\beta(1-\mathbb{E}(\pi|R)) + (T+\tau w)s\nu(q)(1-F(R))(1+\beta\mathbb{E}(\pi|R)) &= 0,\end{aligned} \tag{38}$$

where the last equation is the intertemporal balanced-budget constraint of the government.

The counterpart of program (16) in this economy with taxes is, after imposing the equilibrium result of $w_1 = w_2$,

$$\begin{aligned}\max_{w,R,q,s}\quad & -\phi(s) + s\nu(q)\int_R\Big(v(w(1-\tau)-T) - v(c_u^p) + \beta\pi\big(v(w(1-\tau)-T) - v(c_u^p)\big)\Big)dF(\pi)\\ \text{s. to}\quad & \eta(q)\int_R\Big(\pi y - w + \beta\big(\pi(y-w) - (1-\pi)L\big)\Big)dF(\pi) \geq k\\ \text{s. to}\quad & \tfrac{v(w(1-\tau)-T)-v(z+b-T)}{v'(w(1-\tau)-T)(1-\tau)}(1+\beta R) + Ry(1+\beta) - w(1+\beta R) = 0\end{aligned} \tag{39}$$

Notice that we obtain the non-negative profits condition of the above program by combining the resource constraint of the planner (8) and the balanced-budget constraint of the government (38). That is, the above problem and the planner's problem (9) are the same. Therefore, if there exists a policy $(b, \tau, T, L)$ such that satisfies conditions (36)-(38), then the tax-distorted equilibrium is constrained efficient.

Consider the case of $R^p > 0$. By combining the necessary first order conditions with respect to $q$ and $R$, we can write the former as

$$\begin{aligned}(1-F(R))(1+\beta\mathbb{E}(\pi|R))\Bigg(\bigg(\frac{\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{R}{1+\beta R}\bigg)y(1+\beta) - L\beta\bigg(\frac{1-\mathbb{E}(\pi|R)}{1+\beta\mathbb{E}(\pi|R)} - \frac{1-R}{1+\beta R}\bigg)\Bigg) &=\\ &= \frac{k}{\eta(q)(1-\varphi(q))}\end{aligned} \tag{40}$$

This equilibrium condition coincides with the efficiency condition (25) if and only if $L = 0$.

Given $q^p$ and $R^p$, the equilibrium wage $w$ is determined by the zero-profit condition. Then, the government balanced-budget condition holds at the efficient allocation. Therefore, there are two policy instruments to be determined and two equations to be satisfied, taking into account that $b$ is such that condition (38) holds.

The efficiency condition (23) implies $\frac{v(c^p)-v(c_u^p)}{v'(c^p)} + c_u^p - z + y\frac{R^p(1+\beta)}{1+\beta R^p} - c^p = 0$. Then, plugging conditions (36) and (37) into this expression, and comparing the resulting equation with the tax-distorted equilibrium counterpart of condition (27), we obtain that

$$b = -\tau y \frac{R^p(1+\beta)}{1+\beta R^p} \tag{41}$$

Therefore, $\tau < 0$ if $b > 0$. To isolate the parameter $\tau$, we properly combine equations (36), (37) and (41), and obtain

$$(1-\tau)\left(\frac{Ry(1+\beta)}{1+\beta R} - w\right) = -\frac{v(c) - v(c_u)}{v'(c)},$$

which is the tax-distorted equilibrium counterpart of condition (27). Then, we can determine the remaining policy parameters from equations (36) and (37).‖