

# Labour Share and Productivity Dynamics

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## Abstract

We pose technology shocks where the innovation is biased towards more recently installed plants. On one extreme the shock is like a neutral technological shock, while on the other end it resembles investment specific technological shocks. We embed these shocks in a model with putty-clay technology and estimate it requiring that the model replicates the volatility properties of the Solow residual and the overshooting property of the labour share of output. Our estimates show that putty-clay nature of technology, a time bias towards new plants and competitive wage setting replicate well the overshooting property.

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## 1 Introduction

In this paper we present a new way of modelling aggregate productivity shocks, where we introduce a “time” bias with respect to installed capital: shocks may affect newer firms in a stronger way than older firms. On the one end of the spectrum, when shocks do not exhibit time bias, these shocks are like standard TFP shocks. If the time bias is extreme (only new machines are affected by it), the shocks point to behaviour akin to that of investment specific technological shocks.<sup>1</sup>

To make this notion operational we embed the technology shocks into a model with putty-clay technology where new and old firms have a sharp distinction, as in Johansen (1959), Atkeson and Kehoe (1999), Gilchrist and Williams (2000), Gourio (2011) and Wei (2003). Under this technology assumption, individual productive units consisting of one worker each are indexed by their installation dates and differ in the “quality” (or size of its capital). The pre-installation size menu available to firms is Cobb-Douglas (i.e., ‘putty’), but once installed, plants remain in place without the possibility of changing its quality (i.e., ‘clay’). The putty-clay assumption creates a Leontief productive structure in the short-run between capital and labour: workers must be attached to machines in order for the firm to produce. This is a major departure from the traditional Cobb-Douglas setup, which implies an elasticity of substitution between inputs fixed to one for all time horizons. Since there is dispersion of productivities across plants, we explore both competitive wages and Nash bargained wages as is standard in the search and matching literature.<sup>2</sup>

We use this model to address the cyclical behaviour of the labour share, an issue of importance for different strands of the literature: Gali and Gertler (1999) and Sbordone (2002) relate it to inflation dynamics while Santaaulalia-Llopis (2012) argues that it matters for measurement of the effects of productivity shocks. More specifically, we aim to replicate the overshooting property of the labour share as studied by Choi and Ríos-Rull (2009) and Ríos-Rull and Santaaulalia-Llopis (2010): after a positive technological shock, the share of output that corresponds to labour falls temporarily but it quickly rises, staying persistently above average for around thirty quarters. Consequently, we estimate the process for the shock together with the degree of time bias, targeting not only the standard deviation and persistence of the model implied Solow residual, but also the impulse response function of the U.S.

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<sup>1</sup>Investment specific technology shocks have gained prominence as a source of aggregate fluctuations. The relative importance of these shocks has been studied extensively, using diverse methods. See for example Fisher (2006b), Fernandez-Villaverde and Rubio-Ramirez (2007), Justiniano *et al.* (2010), Justiniano *et al.* (2011), Bai *et al.* (2011), Schmitt-Grohe and Uribe (2011) and the survey by Ramey (2016).

<sup>2</sup>See Pissarides (2000).

labour share, using the method of simulated moments.<sup>3</sup>

We find that putty-clay technologies can match the overshooting property of the labour share. The best estimates are sharp and they point to a high degree of time bias towards new technologies. The exercise also leans toward the existence of competitive labour markets. In our framework, the response of the labour share is closely related with the behaviour of employment, which reacts significantly to technology shocks due to its one-to-one relationship with installed capacity (the *putty-clay* effect): firms can only make use of improved conditions by opening new plants *and* hiring workers for those plants. This effect is further leveraged by the presence of time bias, since firms have added incentives to create employment because of the short-lived nature of shocks under this specification. Furthermore, the increase in employment does not have a negative effect on productivity of other workers and we show that the overshooting is maintained both under competitive wages and Nash bargaining.

The workings of our model are quite different from what happens in setups with Cobb-Douglas technology: there, factor shares move little and any departures of labour share from its steady state value are short lived (see Choi and Ríos-Rull (2009)). The main difference between putty-clay and Cobb-Douglas with putty-putty factors (when shocks are neutral) is the implied elasticity of substitution between inputs in the short run: zero in putty-clay, one in Cobb-Douglas. Hence, the way in which capital and labour can be replaced is key to understand intuitively the overshooting of the labour share in our setting. We show this by comparing simulations of economies with different types of technology, different degrees of bias in the technology shocks and different wage setting mechanisms. We study how they manage to replicate the overshooting property (if at all).

Our findings are related to two important issues in modern macroeconomics: the equilibrium unemployment puzzle and the effects of productivity shocks on the labour input. Strands of the literature following Shimer (2005) on the former and Galí (1999a) on the latter, see these findings as tell tale signs of the importance of wage rigidities in the aggregate. Here we present an alternative framework which shares model features with setups used in both literatures, but does not rely in any form on wage rigidity to produce significant amounts of responses in unemployment and employment.

With respect to the unemployment puzzle, the literature studying models with labour market frictions finds very poor responses of unemployment to realistic shocks (the so called *Shimer* puzzle).<sup>4</sup> The

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<sup>3</sup>Note that our identification relies on the structure we impose on aggregate technology. For a general discussion on the feasibility of identification of both neutral and non-neutral technology shocks, see Diamond *et al.* (1978) and Leon-Ledesma *et al.* (2010).

<sup>4</sup>See Shimer (2005), Hall (2005), Blanchard and Galí (2007), Costain and Reiter (2008) and Hall (2009), among others.

main proposition from this literature is that the terms of bargaining in real wages matter for labour market outcomes during the cycle:<sup>5</sup> incentives for firms to post vacancies during favourable aggregate conditions depend on how much profits they can get from matched employees (i.e., wedge between labour productivity and real wages). Hence, rigid wages give more incentives for employment creation by increasing the dynamic difference between labour productivity and real wages. Beyond the ability of our model to produce high levels of employment and unemployment volatility, our setup has two interesting implications for this literature: First, putty-clay technology is able to produce a delayed response in employment after a technology shock, in a model *without matching frictions*. This is because firms need to invest in units before production and hirings can take place. Second, the confluence of Nash bargaining and time bias mimics rigid wages: new unit wages react to changes in aggregate conditions much more than older units.

With respect to the effects of technology shocks, our results contrast with the literature following Galí (1999b), who uses long run restrictions on a vector autoregression framework and finds a negative effect of productivity shocks on aggregate hours in the U.S. economy. Our model specifications where shocks are more time biased thus are in line with Fisher (2006a), who constructs a model where only neutral and investment-specific technological shocks can have a long-run impact on labour productivity and finds that hours rise in response to technology shocks. Related to this point, Cantore *et al.* (2019) show that both a textbook and a large scale version of the New Keynesian model are unable to predict the behaviour of labour share after a monetary policy shock. This inability mirrors the problems of the standard real business cycle model with Cobb-Douglas technology in explaining cyclical movements of the labour share following technology shocks. We see this as evidence that labour share is an important source of validation for modern macroeconomic models.

Recent papers have studied the cyclical properties of the labour share in the U.S. Shao and Silos (2014) pose a model with frictional labour markets and costly entry of firms and highlight the low correlation of real interest rates and output. León-Ledesma and Satchi (2019) also generate the overshooting property of the labour share in a model where there are adjustment costs to technology choices that generate an elasticity of substitution between capital and labour that is lower in the short run than in the long run.

The structure of the rest of the paper is as follows: In Section 2 we present our model and its

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<sup>5</sup>See Mortensen (1992), Caballero and Hammour (1996) and Rogerson and Shimer (2010).

variants. Section 3 poses and discusses the estimation of the shock process for the various models. Section 4 discusses the performance of the models first in terms of the overshooting property of the labour share and then in relation to the standard business cycle second moments. Section 5 concludes. Various appendices provide additional details.

## 2 The Model

The model is a growth model in discrete time with infinitely lived agents. Production of the unique good of this economy takes place in independent plants that have one worker each and that may differ in the amount of capital installed. We denote this amount as  $k$  and we think of it as the “size” or “quality” of the plant. We assume that workers provide labour inelastically, thus  $k$  also represent capital/labour ratios at the plant level. A plant takes one period to become operational and once built, its capital cannot be changed and its scrap value is zero. Productive units face an exogenous break-down probability of  $\delta$ .

Firms can find workers after paying a hiring cost  $c^v$  that can be thought of as due to vacancy posting and training. This cost partly replicates the hiring frictions in a standard search and matching framework (except for the increased difficulties of finding a worker in expansions). Moreover, given the required “incubation” period for plants, aggregate employment in the economy moves with some lag, as if there were matching frictions (because the capital that the worker will use has to be installed), adding a mechanism to stall the fast expansion of employment right after a positive shock in aggregate conditions. Once a worker is hired her wage is set by bilateral Nash bargaining between the firm (owner of the plant) and (the family of) the worker.

A plant of size  $k$  produces an amount  $e^z k^\alpha$  of goods per period, where  $z$  is an aggregate productivity shock. Its evolution is given by

$$z = \begin{cases} \tilde{z} & \text{if new machine,} \\ \lambda \tilde{z} & \text{otherwise,} \end{cases} \quad (1)$$

with

$$\tilde{z}' = \rho \tilde{z} + \epsilon', \quad (2)$$

where  $\rho \in (0, 1)$  and  $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$ . *New machine* denotes productive units which were installed one period ago and will start production in the current period. Note that when  $\lambda = 1$ , the shock reverts to a classic factor neutral aggregate shock, which in our case, it could be understood also as machine-

cohort-neutrality. On the other extreme, if  $\lambda = 0$ , the shock affects only the newest productive units installed which makes it look similar to an investment specific technical shock (in Appendix A we provide a discussion of how the two shocks are related).<sup>6</sup>

The aggregate state of the economy  $S$ , is summarized by the aggregate shock  $\tilde{z}$  and  $X(k, t)$ , the measure of installed plants of capital amount  $k$ , and age or type  $t \in \{0, 1\}$ , one if it is a new plant and zero otherwise.

## 2.1 Households

The economy is populated by a unit measure of households (families), each of them consisting of a mass of identical consumers/workers (normalized to one). Each household has preferences over consumption and leisure streams of their members. Every household member is part of the labour force, but employment is limited to the number of machines operating in the economy. We assume perfect capital markets, so there is no distinction in terms of consumption between employed or unemployed household members.

We write the per period utility function of the family as  $u(c) + v(n) = \log(c) + (1 - n)b$ , where  $c$  is the common consumption level for everyone inside the family,  $n$  is the fraction of household members that work this period and  $b$  is the additional value of leisure to the non-working members of the family (we normalize the leisure value to those working at zero). In each family, its working members do so in a variety of firms or plants, each one characterized by size  $k$  and type  $t$ . Family employment then satisfies  $n = \sum_{t \in \{0,1\}} \int x(k, t) dk$ , where  $x$  is the size/type distribution of firms where the family members are employed at. In addition, households discount the future at rate  $\beta$  and take the appropriate expectations whenever necessary. The state of the household (which we denote by  $s$ ) is comprised also of its assets, so  $s = \{a, x\}$ . The problem of the household in value function form requires us to also specify the

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<sup>6</sup>We thank Joachin Hubmer, who pointed inaccuracies in our comparison to investment specific technological shocks in a previous draft of our paper.

evolution of the aggregate state  $X$ . The value function is then

$$V(S, s) = \max_{c, a'} \log(c) + (1 - n)b + \beta E\{V(S', s')\} \quad \text{s.t.} \quad (3)$$

$$c + a' = R(S) a + \sum_{t=\{0,1\}} \int w(k, t, S, s) x(dk, t) + \pi(S), \quad (4)$$

$$x'(k, 1) = M(S, x), \quad x'(k, 0) = (1 - \delta) \sum_{t=\{0,1\}} x(k, t) \quad \forall k, \quad (5)$$

$$X'(k, 1) = M(S, X), \quad X'(k, 0) = (1 - \delta) \sum_{t=\{0,1\}} X(k, t) \quad \forall k. \quad (6)$$

Here  $R(S)$  is the aggregate gross rate of return,  $w(k, t, S, s)$  is the real wage a household member can command in a plant with characteristics  $(k, t)$ ,  $\pi(S)$  represents current period profits of firms owned by the household,  $M(S, x)$  is the measure of hires from the unemployed pool of an individual house (with characteristics  $x$ ) into a  $(k, 1)$  firm and  $M(S, X)$  is the aggregate measure of the unemployed (defined by  $X$ ) going to the same type of firms.<sup>7</sup> The last two equations in the problem of the household above, show that households need to forecast the measure of employment in different types of firms. The optimization of the household involves the standard Euler equation

$$1 = \beta E \left\{ \frac{c(S, s)}{c(S', s')} R(S') \right\}. \quad (7)$$

## 2.2 Firms and Investment Decisions

The value of an active plant of size  $k$  and type  $t$ , attached to a worker from a household with characteristics in  $s$  is given by<sup>8</sup>

$$\Pi(k, t, S, s) = e^z k^\alpha - w(k, t, S, s) + (1 - \delta) E \{Q(S') \Pi(k, 0, S', s')\}, \quad (8)$$

where  $w(k, t, S, s)$  is the wage paid to the worker attached to this firm, and  $Q(S')$  is the discount rate for the firm,  $Q(S') = [R(S')]^{-1}$ . Once a plant is installed, the firm does not make any decisions: plants get destroyed at the exogenous rate  $\delta$  and their productivity is given by  $z$  (depending whether they are new or old) and the installed capacity  $k$ .

<sup>7</sup>Note that we use notation  $M(S, x)$  and  $M(S, X)$  to explicitly show how hiring depends only the number of unemployed.

<sup>8</sup>With this notation, we are assuming that firms and workers get matched before the investment decision. As we show in the next section, in equilibrium, this timing is irrelevant.

We can define the value of marginally increasing plant size  $k$ :

$$\Pi_k(k, t, S, s) = e^z \alpha k^{\alpha-1} - w_k(k, t, S, s) + (1 - \delta) E \{Q(S') \Pi_k(k, 0, S', s')\}, \quad (9)$$

This expression is useful to calculate the optimal installation size, which solves the following problem

$$\max_k -k + E \{Q(S') \Pi(k, 1, S, s)\}, \quad (10)$$

where the solution  $k^*(S)$  satisfies the first order condition

$$1 = E \{Q(S') \Pi_k(k^*(S), 1, S, s)\}. \quad (11)$$

On the other hand, the number of new plants  $M(S, X)$  that are installed and hire unemployed members from measure  $X$  is determined by the zero profit condition that includes the cost  $c^v$  of hiring a worker:

$$k^*(S) + c^v = E [Q(S') \Pi(k^*(S), 1, S, s)] \quad (12)$$

$M(S, X)$  and  $k^*(S)$  determine overall capital investment (by this type of match) in the current period, since total investment equals  $Mk^*$ .

### 2.3 Wage Determination

In putty-clay economies there is no operational notion of marginal productivity of labour that is a natural candidate for the wage because the ratio of workers to machines is fixed. Yet, households do have a marginal rate of substitution that is equated to the wage and new firms have a zero profit condition that disciplines their behaviour allowing for a competitive determination of the wage. The natural alternative to competition is a bargaining mechanism between workers and firms. Moreover, the competitive wage turns out to be the wage that results when the bargaining weight of workers is zero. The bargaining process between workers and firms requires two additional specifications. Whether it is done within each firm or whether it is done economy-wide,<sup>9</sup> and what occurs in case of breakup of the negotiations (whether the match is dissolved or whether the firm and worker are still attached the following period). We explore both competitive and bargained-for wages. As we use North American

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<sup>9</sup>The latter is mostly used in Northern European contexts. See for instance Krusell and Rudanko (2016).

data, we pose decentralized bargaining. The standard assumption in the literature when wages are bargained for is to assume that in the absence of agreement production within the period does not happen *and* the match is broken, i.e. the worker becomes unemployed and the plant loses the worker.<sup>10</sup> Instead, we assume that in the absence of agreement, production within the period does not occur but the worker and the firm are still matched so the following period a new round of bargaining can take place. This choice requires a discussion. In models where all jobs are identical, the bargaining problem under match breakup that solves for the wage collapses to a simple form because both the worker and the firm when matched in the future will have partners that are identical to those that they are matched with in this period. In a putty-clay economy, however, this is not the case: if negotiations are not successful and the match breaks, the unmatched firm faces a different prospect than all the new firms that will be available next period because they are of a different size. Thus, the standard assumption of the breaking of the match would yield a bargaining problem that depends on the distribution of existing firms and the continuation values of both workers and firms. Our assumption about the bargaining process avoids these complications and make the resulting wage equation similar to the one in the standard Mortensen-Pissarides framework, a feature to which we will return later.

If we assume that the match survives then, the value of the worker for the firm is  $e^z k^\alpha - w(k, t, S, s)$ , while the value of the job for the household is just its wage net of the value of being unemployed,  $w(k, t, S, s) - b c(S, s)$  (in units of consumption).<sup>11</sup> Firms and workers bargain over the match surplus, and if the bargaining power of the household is  $\mu$ , the wage is just fraction  $\mu$  of the surplus which is

$$w(k, t, S, s) = \mu e^z k^\alpha + (1 - \mu)b c(S, s). \quad (13)$$

Given this wage function, plant specific profits each period are given by  $(1 - \mu)[e^z k^\alpha - b c(S, s)]$ . This expression can be used to rewrite Equation (11). Note that in this setting, all firms have the same information and installation costs are the same for them. Thus, only one size  $k^*$  is installed each period.

**Wages in Standard Search and Matching Models** In standard search and matching models the expression for wage setting under bargaining differs for two reasons.<sup>12</sup> One is the aforementioned

<sup>10</sup>See for instance Andolfatto (1996) or Choi and Ríos-Rull (2009)). Christiano *et al.* (2016) pose a model with alternating offers and random breakdowns at a rate that they estimate.

<sup>11</sup>In terms of “utils”, the value of an additional worker for the household is given by  $w(k, t, S, s)u_c(S, s) - b$  where  $u_c(S, s)$  is the marginal utility of consumption. Given log preferences, transforming this expression into units of consumption gives the equation in the text.

<sup>12</sup>See Andolfatto (1996) and Merz (1995).

assumption of the breakdown of the match in case of unsuccessful negotiations that yields an additional term that is weighted by the workers' weight,  $\widehat{c}^v \frac{V}{1-N}$ , where  $\widehat{c}^v$  is the cost of posing a vacancy and  $V$  is the number of vacancies posted. Formally, we have (details are in Appendix B),

$$w = \mu \left[ \alpha e^z K^\alpha N^{-\alpha} + c^v \frac{V}{1-N} \right] + (1-\mu)C b \quad (\text{Match does not Survive}), \quad (14)$$

$$w = \mu \left[ \alpha e^z K^\alpha N^{-\alpha} \right] + (1-\mu)C b \quad (\text{Match Survives}). \quad (15)$$

The other reason for the difference in the wage relative to standard search and matching models has to do with the value of the worker for the firm. With Cobb-Douglas putty-putty technology a worker is worth its marginal productivity, while in the putty-clay technology a worker means some output while not having a worker means no output at all. This accounts for the absence of the term  $\alpha N^{-\alpha}$  in our wage setting equation. Because employment in steady state is close to 1 the main difference between Cobb-Douglas putty-putty and putty-clay economies for wage setting is the existence of the parameter  $\alpha$  in the wage formula of the former. The particular values for the bargaining weights depend not only on the nature of the technology but also on the calibration targets that we set.

## 2.4 Aggregation and a simplified problem

This model can be simplified dramatically (following Gourio (2011)) to get rid of the measure  $X(k, t)$  of plants and substitute it by a few aggregates. Define aggregate installed capacity by type 0 (old) firms as

$$\bar{Y}^0 = \int k^\alpha X(dk, 0). \quad (16)$$

In a similar same vein, capacity of new firms the following period is

$$\bar{Y}^{1'} = M (k^*)^\alpha. \quad (17)$$

These two expressions give a recursive characterization that allows us to avoid using the measure of firms:

$$\bar{Y}^{0'} = (1-\delta) \left[ \bar{Y}^0 + \bar{Y}^{1'} \right]. \quad (18)$$

Given the shock process described in Equation (1), total output is given by

$$Y = e^{\lambda \bar{z}} \bar{Y}^0 + e^{\bar{z}} \bar{Y}^1. \quad (19)$$

Aggregate employment can also be defined recursively with the aid of the number of new plants

$$N' = (1 - \delta)N + M. \quad (20)$$

Similarly, the entire wage bill for the economy is just

$$\omega = \mu Y + N(1 - \mu) b C. \quad (21)$$

Note that the aggregate feasibility constraint in the economy is now written as

$$Y = C + M [k^* + c^v]. \quad (22)$$

In a steady state, employment becomes  $N = M/\delta$ , while output simplifies to  $Y = (M/\delta) k^\alpha$ . Since the capital stock is defined as the number of operational machines (which is the same as the level of employment), this implies that in a steady state

$$\begin{aligned} Y &= \left(\frac{M}{\delta}\right)^{1-\alpha} \left(\frac{M}{\delta} k\right)^\alpha \\ \Rightarrow Y &= N^{1-\alpha} K^\alpha \end{aligned} \quad (23)$$

where  $K$  is total capital. Thus, the aggregate technology in the long run is Cobb-Douglas. This is important for a couple of reasons: In the long-run, the properties of the economy are guided by the usual notions applied to technology in RBC and New Keynesian models under this type of technology. Second, even if in the short run the production technology is Leontief (fixed proportions), the elasticity of substitution is higher (equal to one) in the long run.

Given these simplifications, the aggregate state of the economy reduces to  $\{\bar{z}, \bar{Y}^0, \bar{Y}^1, N, \}$ . The nature of the shock process that distinguishes between new and old installed capacity requires us to add one state variable to those posed by Gourio (2011). The model in turn simplifies to a set of equilibrium

equations, which we solve using standard approximation methods. These equilibrium conditions can be summarized as the Euler equation of the household, the equations for optimal current plant size and optimal number of plant installations by firms and aggregate market consistency: Equation (7), Equations (10) and (11) and Equations (2), (13), (18), (20) and (22) respectively.

The state space for the representative household also collapses, since similar arguments can be used to show that average employment and wage bills suffice to solve the problem of the household. Savings and consumption decisions depend on aggregate interest rates and total labour earnings that the household receives.

### 3 Mapping the Model to U.S. Data

We explore two model economies that differ in the wage setting process. We look at Nash bargaining and at competitive wages (where we set the bargaining power of workers  $\mu$  to zero).

We set the model period to a month, short enough to accommodate employment flows but we aggregate to quarterly frequency for a direct comparison with U.S. data moments.<sup>13</sup> We set  $\beta$  to 0.9966 so that the interest rate is four percent per annum. We set another group of parameters,  $\{b, \delta, \alpha, c^v, \mu\}$ , (value of home production, firm destruction rate, coefficient of capital in the production function, hiring cost, and worker's bargaining weight) to match a set of long-term statistics for the US economy. Although these parameters affect jointly the numerical steady state of the model, there are close links between individual parameters and model predictions, which we use to calibrate this set. The destruction rate of plants  $\delta$  is linked closely to aggregate capital and savings, thus we use the aggregate consumption-output ratio (0.75). The coefficient in the production function  $\alpha$  is used to match the average value of the labour share, 0.6514;<sup>14</sup> the cost of posting a vacancy  $c^v$  is chosen to match a flow cost of a vacancy in terms of days of pay per hire of 0.446 which closely matches an aggregate vacancy expenditure relative to annual GDP of 0.50%, reported by Cheron and Langot (2004). This is close to the value of 0.433 used by Hall and Milgrom (2008) based on Silva and Toledo (2009). The bargaining weight of workers is related to the unemployment rate, which is 0.058 in the US during our sample. Finally, in the Nash bargaining economy we set the value of home production  $b$  to match a total value of leisure per period that amounts to 0.70 of household consumption (as calculated by Hall

<sup>13</sup>We solve for each economy using perturbation methods. We use the free package *Dynare*, version 4.5.7 in Matlab R2018a. We then simulate each economy for 33,000 months, discard the first 3,000 observations and then create quarterly series by simple averaging of the monthly data.

<sup>14</sup>We use the value reported by Koh *et al.* (2020).

and Milgrom (2008)).<sup>15</sup>

Competitive wage setting implies that workers have no bargaining power ( $\mu = 0$ ), which implies that in this economy the value of leisure in terms of consumption becomes exactly 1.

Table 1: **Steady State Parameter values**

Parameter	Description	Nash Bargaining	Competitive Setting
$b$	value of leisure	0.6459	0.9226
$\delta$	plant destruction rate	0.0084	0.0084
$\alpha$	production curvature	0.5383	0.3449
$c^v$	vacancy cost	0.3090	0.3083
$\mu$	workers bargaining weight	0.3596	0.0000

The implied values for the parameters are reported in Table 1. While most parameters are in range of numbers used in the literature, the main departure is in parameter  $\alpha$  in the Nash bargaining economy, which differs from the standard value of  $1/3$ . This deviation is due to the way bargaining operates in putty-clay environments. On the other hand, the plant destruction rate is close to the value implied by establishment statistics.<sup>16</sup>

### 3.1 Estimating the Productivity Shock Process

To estimate the model we use a combination of moments from aggregate US time series (a detailed description of the data used and the methodology is in Appendix C). As noted above, we use empirical moments coming from quarterly frequency, which we match by aggregating our monthly model.

We estimate the parameters controlling the process for the shock,  $\{\rho, \sigma_\epsilon, \lambda\}$ , using simulated method of moments. Two of the moments that we use are obtained from an AR(1) representation of the Solow residual which we estimate following closely the methodology in Ríos-Rull and Santaaulalia-Llopis (2010): Using aggregate data from output, capital and employment, we construct and linearly detrend the Solow residual. The autocorrelation and estimated variance from the AR(1) are two moments which are related to parameters  $\rho$  and  $\sigma_\epsilon$ .<sup>17</sup>

<sup>15</sup>In their discussion this value amounts to their value of  $z$  and includes not only the value of unemployment but also the direct value of leisure.

<sup>16</sup>Using information from <https://www.census.gov/programs-surveys/bds/data/data-tables/legacy-firm-characteristics-tables-1977-2014.html> we compute a monthly establishment exit rate of 0.0089.

<sup>17</sup>In fact, they are identical to  $\rho$  and  $\sigma_\epsilon$  in the case of quarterly RBC models with putty-putty Cobb-Douglas technology and competitive factor pricing, but not in putty-clay models or in monthly models.

More importantly, we also target the dynamics of labour share using its impulse response function from a bi-variate vector autoregression of order one between linearly detrended (log) labour share and linearly detrended (log) output. Our rationale for this is as a way of dealing with the secular decline of the labour share in the last few years. Below we extend our analysis to not detrending the labour share. The main facts also hold with some nuances.

From the VAR(1) estimates we extract the orthogonalized impulse response of labour share to a shock in the output equation, which we use as moments to be matched by simulated data of our model. Appendix D presents a discussion of our estimated impulse responses (see the right panel of Figure 6) and some robustness checks. Note that, contrary to the literature using structural VARs and some form of long run or sign restriction to identify structural shocks, we are not inferring any type of causality from our VAR estimates. Rather, we use these as moments to be matched by our model. Our methodology is similar to the estimation procedure in Christiano *et al.* (2005), who estimate a set of parameters to match empirical impulse response functions. Below we also compare the predictions of the model in terms of impulse responses of other aggregate variables. The procedure we follow for those is analogous to the one we take for the labour share: we estimate bivariate vector autoregressions between detrended output and the detrended aggregate of interest and retrieve from that estimation the response of the variable to a shock in the output equation.

Our estimation algorithm is standard: for each proposed economy, we choose model parameters to solve and simulate model data, from which we apply the same empirical methodology to obtain model counterparts to the data moments described above. As we want to effectively replicate the standard deviation and the autocorrelation of the Solow residual as close as possible across the different economies, we pose very large weights on those two moments relative to those of the impulse response of the labour share (a factor of 1,000), which make these moments almost exactly equal in the model and in the data.

The estimated parameters are in Table 2 where the standard errors are computed using standard methods.<sup>18</sup> We compare the estimates that we obtain with those that result from imposing  $\lambda = 1$ , that is, economies subject to standard neutral technology shocks. We see how small are the values obtained for  $\lambda$ : close to zero with Nash bargaining and 0.22 for competitive wages, pointing to a time

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<sup>18</sup>We follow Duffie and Singleton (1993): the standard errors are the square roots of the diagonal elements of matrix  $\{G'SG\}^{-1}$ , where  $G$  is the matrix of partial derivatives and  $S$  is a diagonal matrix with the inverse of the variance of the data moments in its diagonal.

Table 2: **Putty-Clay Economies:**  
Estimated Monthly Process for the Technology Shock

	Nash Bargaining		Competitive Setting	
	Time Bias	No time bias	Time Bias	No time bias
$\lambda$	0.0051 (0.001)	1. –	0.2181 (0.126)	1. –
$\rho$	0.9731 (0.003)	0.9798 (0.007)	0.9835 (0.002)	0.9835 (0.007)
$\sigma_\epsilon$	0.4081 (0.035)	0.0055 (0.000)	0.0245 (0.013)	0.0055 (0.000)
Sum Square Residuals of Labour Share IRF	1.58E-05	6.88E-05	4.35E-06	5.54E-06

Standard errors in parenthesis. The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

biased productivity shock rather than to a neutral one. Since we require the model to match the overall volatility of the economy (in the form of implied Solow residual behaviour), a consequence of the strong concentration of the productivity increases in fewer plants, the estimates for the standard deviation of the innovations  $\sigma_\epsilon$  are quite large for both the Nash bargaining economy and the competitive wage economy when compared to their *No time bias* counterparts.

We have a natural measure of fit of the estimates, the sum of the square of the residuals for impulse response moments between US and model generated data that are also displayed in the table. From the table we see that the competitive setting yields better estimates than the Nash bargaining ones and that time bias implies smaller errors.

These findings show that in our setup, using a different identification strategy than what is usually the case, we find strong evidence that the nature of the stochastic process for productivity seems to be more related to new plants than to established plants. Secondly, our baseline estimation favours competitive wages. The earlier result is quite robust to different treatments of the data (see Appendix D), while (as we show below) the competitive wage model fits better untargeted components behind the overshooting of labour share.

## 3.2 Robustness

We turn to see whether our findings are robust to the length of the period (monthly versus quarterly) and to the recent downward trend in the labour share. In Appendix D we also look at overshooting property when we use long run restrictions to overidentify the technology shocks and when we use Fernald's adjusted TFP series.

**The Length of the Period** Recall that the model is monthly and the targets (because of data collecting periodicity) are quarterly. While the mapping between versions of the growth model with different length periods is typically straightforward, a shock with time bias introduces nuances in this mapping because it concentrates productivity gains in the most recent period. As such, there is no analytical relation between a quarterly time-biased shock and a monthly one. If time bias were to be an intrinsically monthly phenomena, a way to think of it within a quarterly model would be as having only one third of the newly created plants receive the time biased shock level while the rest are subject to the same productivity level than older plants. For these reasons, we have estimated the shocks process for a quarterly version of the model and we display the estimates in Table 3. The findings are reassuring: the variance of the quarterly model is smaller as the time bias affects a larger set of plants. In the same vein, the persistence is smaller. The time bias changes very little being almost complete in the Nash bargaining wage setting and a bit larger than in the monthly version in the competitive setting. These estimates are completely consistent with those of the monthly model which is more appropriate when thinking of employment flows.

**The Trend in the Labour Share** In recent years the labour share of output has experienced an important decline which is well documented by Karabarbounis and Neiman (2014), Elsby *et al.* (2013), Koh *et al.* (2020) and De Loecker *et al.* (2020) to name a few. Above we have dealt with this problem by detrending the labour share avoiding this non-cyclical pattern. To explore the extent to which this may affect our findings, we have also estimated the process for the technology shock without detrending the series and with a shorter sample period, where the overshooting property was first noticed by Ríos-Rull and Santaaulalia-Llopis (2010), and which excludes the more recent years of diminishing labour share (1954:Q1 to 2004:QIV). Figure 7 in the Appendix shows the impulse responses of the labour share for all these definitions and we can see how the overshooting property is present irrespective in all of them. If anything, the longer sample implies more overshooting. On the other hand, detrending, while reducing

Table 3: **Putty-Clay Economies:**  
Estimated Quarterly Process for the Technology Shock

	Nash Bargaining		Competitive Setting	
	Time Bias	No time bias	Time Bias	No time bias
$\lambda$	0.0017 (0.003)	1. –	0.1245 (0.031)	1. –
$\rho$	0.9348 (0.008)	0.9553 (0.014)	0.9648 (0.006)	0.9649 (0.0123)
$\sigma_\epsilon$	0.2774 (0.021)	0.0078 (0.000)	0.0531 (0.010)	0.0078 (0.000)
Sum Square Residuals of Labour Share IRF	3.82E-06	7.11E-05	1.94E-06	1.96E-05

Standard errors in parenthesis. The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

a bit the overshooting, makes it happen a bit faster.

When we re-estimate the technology shock with these alternative impulse responses (Tables 12 and 13) we see consistent evidence of a large time bias, with the values of  $\lambda$  for the Nash bargaining wage setting economies ranging from .01 to .07 and with those of competitive wage setting between .2 and .5 (all except one are below .3). The better performance of the competitive wage setting economy relative to the Nash bargaining one is no longer the case in terms of the sum of the square of the residuals: for two of the four impulse responses the residuals are larger in the competitive economies and for the other two, they are larger in the Nash bargaining economies. However, the joint behaviour of consumption and output in the competitive economies is more similar to the data than that of the Nash bargaining economies. In particular, all competitive wage economies present cyclical correlations (with respect to output) which are much closer to the data than Nash bargaining economies, making us think that the former are a better description of US economy.<sup>19</sup>

<sup>19</sup>See Appendix D for details.

### 3.3 Economies with Cobb-Douglas (Putty-Putty) Technology

To understand the role of the different elements, we also explore standard search and matching economies with Cobb-Douglas putty-putty technology (where capital and labour are fully transferable across plants). To get further insight in the role of wage setting we pose three versions differing in the wage setting. One with dynamic wage bargaining, the standard in search and bargaining environments,<sup>20</sup> where unsuccessful negotiations result in the breaking up of the match. Another economy where the match survives the break up of negotiations like in our putty-clay economy (*Static Nash Bargaining*). Finally, one where the wage is determined competitively. Details of this model and its functional forms are in Appendix B.

Table 4: **Cobb-Douglas putty-putty Economies** (standard technology)

	Dynamic Nash Bargaining	Static Nash Bargaining	Competitive Setting
Steady State Parameters			
$b$	0.644	0.612	0.922
$\alpha$	0.343	0.362	0.343
$\omega$	0.489	0.468	0.487
$c^v$	0.087	0.085	0.086
$\mu$	0.691	0.971	0.000
$\delta$	0.008	0.005	0.008
Estimated Process for the TFP Shock Standard errors in parenthesis			
$\rho$	0.981 (0.007)	0.980 (0.007)	0.981 (0.007)
$\sigma_\epsilon$	0.0055 (0.000)	0.0051 (0.000)	0.0055 (0.000)
Sum Square Residuals of Labour Share IRF	2.21E-04	2.24E-04	2.13E-04

The last row displays the sum of square of the residuals between the impulse response function of the data and each model (given the weights that we use for the errors of the Solow residual statistics these are zero in all cases. See Appendix C).

The steady states of the putty-putty economies are the same as those of the putty-clay economies. We target average labour share (0.6514), the consumption output ratio (0.75), an unemployment rate of (0.058), the value of leisure (0.7) and the flow cost of vacancies (0.5% of GDP). Because these

<sup>20</sup>See Merz (1995), Andolfatto (1996).

economies also have search and matching frictions, some additional targets are required: market labour tightness is set to 1, the elasticity of the matching function with respect to vacancies ( $\psi$ ) to 0.6 and the separation rate ( $\chi$ ) to 0.03 (See Shimer (2005) for a discussion of these targets). We estimate the shock process of these economies exactly like the putty-clay ones.

The results are in Table 4. Interestingly, we see that the static Nash bargaining worker's weight is larger than the dynamic one while the competitive value is (by definition) zero. This indicates that the dynamic Nash bargaining process is somewhere in between the static and the competitive process.

One thing to note is that the estimates of the shock process are essentially identical for all economies: the details of the wage setting do not affect in any relevant way how well the overshooting of the labour share is replicated. In fact, these economies do quite badly with errors that are over one order of magnitude larger than those of the putty-clay economies. Why this is the case is the question that we address in the next Section.

## 4 Properties of the Estimated Model Economies

We start discussing the logic of why the model economies with putty-clay technology are able to deliver the overshooting property of labour share in Section 4.1. We then move to estimate the shock directly on employment and the consumption to output ratio in Section 4.2. We finish the section describing the standard business cycle properties of these economies in Section 4.3.

### 4.1 The Overshooting Property of Labour Share

How do the model economies manage to match the behaviour of the labour share? Recall that after an initial positive technology shock, labour share drops but subsequently exhibits a positive deviation from its long run trend, which is larger than the initial drop.

To see what are the determinants of labour share we take Equation (13), integrate over plants and divide by output to get an expression for the aggregate labour share:

$$LSH = \mu + (1 - \mu) b N \frac{C}{Y}. \quad (24)$$

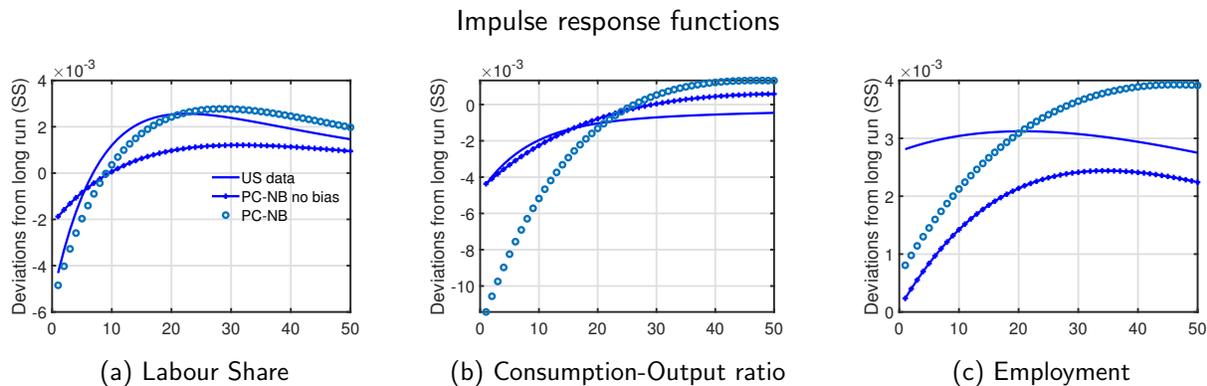
Note first that because we calibrate all the economies to the same values for the labour share, employment rate and consumption to output ratio in steady state, the value of  $b$  varies with the value of  $\mu$  to satisfy this equation. In particular, from Table 1 we have  $b \sim .65$  and  $\mu \sim .36$  implying a

slope  $(1 - \mu)b$  of .42 for the Nash bargaining wage setting economy and  $b \sim .92$  and  $\mu = 0$  implying a slope of .92 for the competitive economy. Now note that, given equation (24) and these implied slopes, movements in employment and the consumption to output ratio have much *stronger* implications for the labour share in the competitive economy than in the Nash bargaining one. This helps explain why the former has an easier time (in terms of the size of the required time bias parameter) in achieving the overshooting property.

Note also that this affine relation between the labour share and the product of the consumption to output ratio and employment is *not* an accounting identity, but an equilibrium relation and hence it does not necessarily hold as such in the data.

**Putty-Clay Nash Bargaining Economies,  $\mu > 0$ :** Figure 1 shows the impulse response functions of labour share, the consumption to output ratio and employment of the putty-clay economies with Nash bargaining wage setting with and without time biased technical change. Recall that the time biased technical change implied a time bias of 99.5% relative to the neutral one (in the sense that the estimated  $\lambda$  is 0.51% while the neutral is 1). Recall also that the sum of the square of the residuals of the neutral technology shock is more than four times larger than in the economy where the shock has time bias.

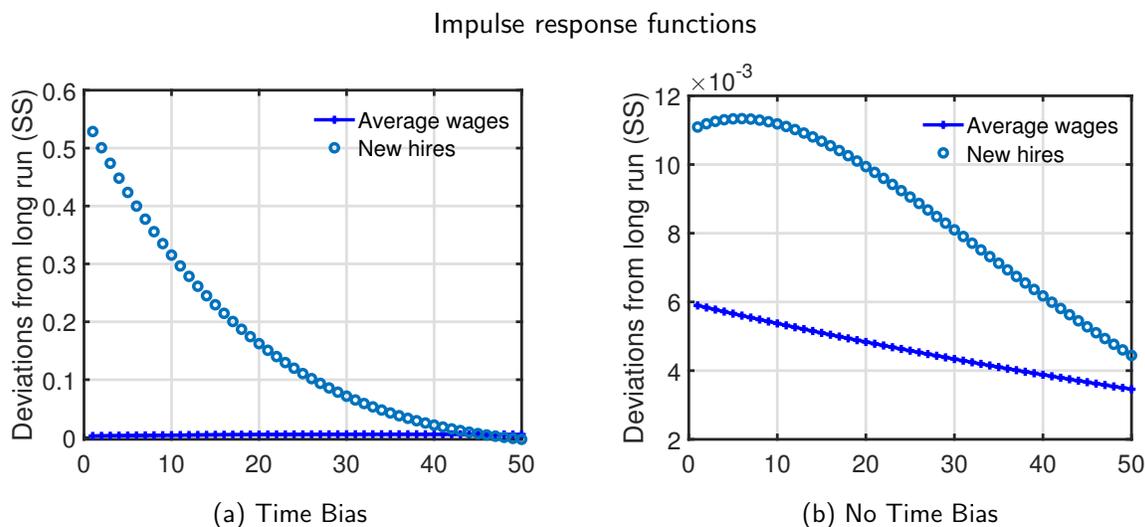
Figure 1: Putty-Clay Economies with Nash Bargaining Wage Setting with and without Technology Shock Time Bias



The left panel of the figure shows that the economy with a time bias technology shock does a reasonably good job of generating the overshooting property of the labour share. It has the right amplitude even though it is slightly delayed. The no time bias shock manages to get only a bit of the overshooting property but much less than that in the data. When we look at how the model economies

behave in terms of the components of the labour share the results are not well aligned with the data. The low slope of the relation between the labour share and the product of employment and the consumption to output ratio imply that these have to move a lot to replicate the labour share. First negatively and then positively which results in an exaggerated fall in the consumption to output ratio that slowly vanishes to the point of changing sign (in fact it generates a countercyclical consumption behaviour, as seen in Section 4.3). To complete the picture the response of employment is much weaker at the beginning, but accumulates over time yielding a higher response than the data at the five year mark. All of this is achieved with an extreme form of time bias. The economy with no time bias cannot generate enough of an overshoot: the consumption to output ratio does not fall that much and employment responds quite slowly

Figure 2: Wage Responses in Putty-Clay Economies with Nash Bargaining with and without Time Bias



To better understand the response of employment in these two economies, Figure 2 shows the response of average wages of new and established plants in both Nash bargaining economies.<sup>21</sup> The results are striking: the extreme time bias of the technical shock together with the decentralized nature of bargaining makes for an enormous wage increase in new plants. While we do not have data on this variable, casual observation indicates that this is unlikely to be the case (we would expect to see a lot of workers switching from old to new plants in expansions).

<sup>21</sup>Because of the decentralized Nash bargaining, these are the only economies that display wage dispersion.

**Putty-Clay Economies with Competitive Wage Setting,  $\mu = 0$ :** Figure 3 shows the impulse response functions for putty-clay economies with competitive wage setting with and without technical time bias. They both match the impulse response of the labour share better than the economies with Nash bargaining. In fact, even the competitive economy without time bias does better than the Nash bargaining economy with time bias. Comparing the two competitive economies we see that the one without time bias is a bit worse (it has 27% higher sum of square residuals).

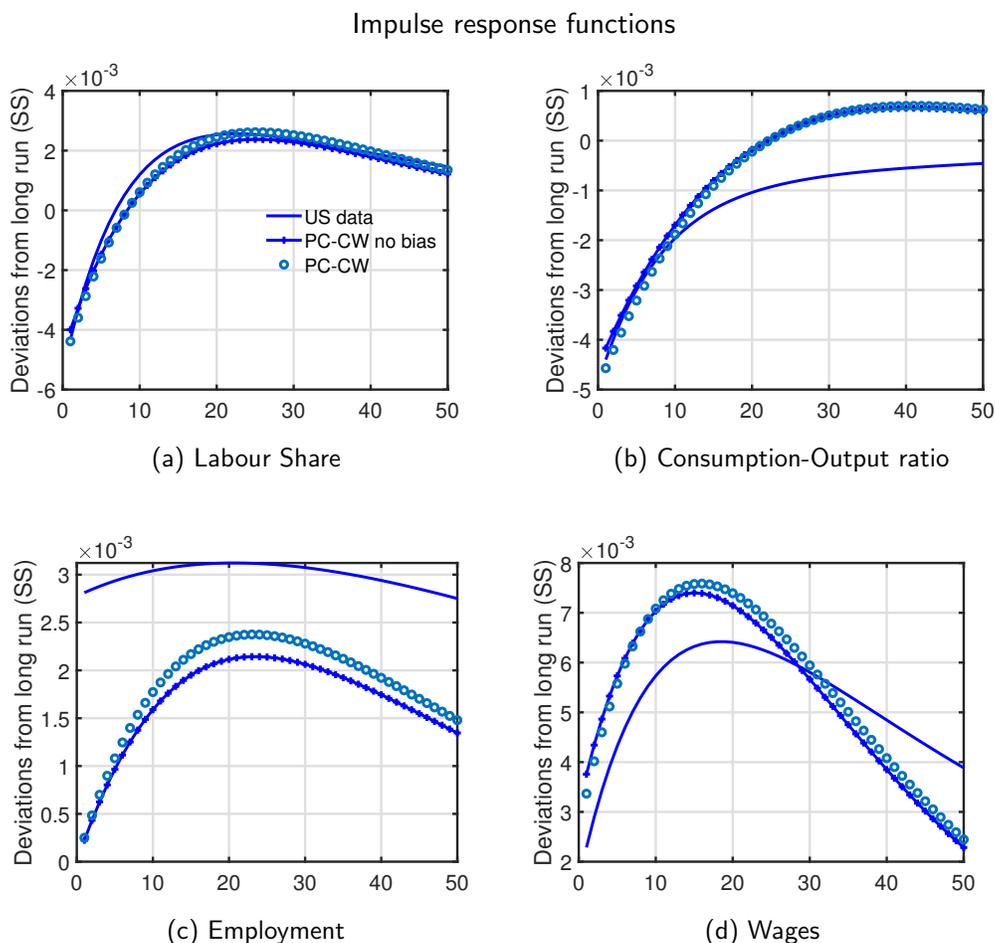
In terms of the determinants of labour share we see (panel (b)) that the consumption to output ratio of both economies are similar and their responses exhibit more amplitude than the US data, while matching well the first ten quarters after the initial shock. Both economies display a similar pattern in terms of employment creation (panel (c)): lower than in the data, with time bias being able to produce a more pronounced response between periods 10 and 30 after the initial shock. Recall that the equation that determines the behaviour of labour share in terms of employment and the consumption to output ratio is an equilibrium equation that holds in the model but not necessarily in the data, where other concerns, such as hours per worker and the participation margin (to name a few) may be quite relevant.<sup>22</sup>

Both models also have a reasonable impulse response of wages, around 20% more than in the data at its peak. Overall, we can see that competitive wage setting with a productivity shock biased towards new technologies accounts well for the overshooting property of labour share even if the response of employment is not strong enough.

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<sup>22</sup>In particular, since we do not model the participation margin and consider only employed and unemployed individuals, our model is restricted in terms of employment creation.

Figure 3: Putty-Clay Economies with Competitive Wage Setting with and without Technical Time Bias



**On the role of time bias.** In putty-clay economies, all employment creation requires new investment. Time bias ( $\lambda$ ) allows for a concentration of the effects of the productivity shocks in new plants where they can attract employment creation as opposed to what occurs in older plants where the shock acts as a rent to be shared by workers and plant owners under Nash bargaining (or just by plant owners under competition). Time bias then amplifies the responses of employment and labour share to shocks, as can be seen in Figure 1 and Figure 3. When technological shocks are extremely time biased towards new machines, firms have added incentives to create employment to seize better aggregate conditions. Also, this is aided by the fact that the specifications with low values of  $\lambda$  go hand in hand with high levels of aggregate shock volatility: if only the newest machines can make use of TFP shocks, and the

model has to be consistent with moments of the Solow residual in US data, then the estimation will lead to higher values for  $\sigma_\epsilon$  which in turn generate higher levels of employment after positive shocks.

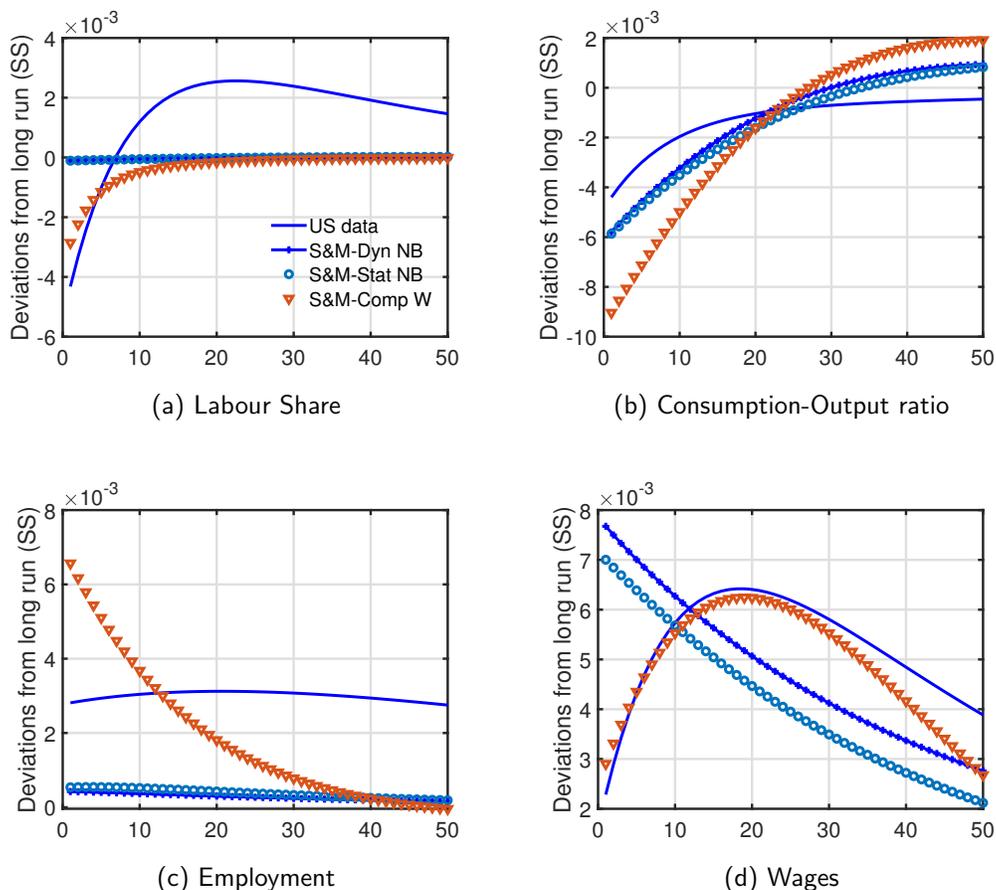
In economies with Nash bargaining wage setting, time bias has also a differential effect on wages, as seen in Figure 2. When only the newest machines are affected by technological shocks, average wages become sluggish to innovations in technology, since most productive units only see their wages react to aggregate conditions through movements in consumption (outside option of workers). This effect is interesting in that mimics “fixed” average wages in an endogenous way which also helps the model in producing more volatility of employment by altering the benefits of hiring workers: wages of new workers are relatively high only one period, while their productivity is long lasting, tied to their machine size  $k$ . However, in the estimated economy the differences are extreme given our simplifying assumption of the time bias operating only one period.

**Putty-clay versus Cobb-Douglas putty-putty Technologies** In Choi and Ríos-Rull (2009) is well documented the ill-performance of search and matching Cobb-Douglas putty-putty economies in replicating the overshooting property of labour share. Comparing those economies with the putty-clay economies at the centre of this paper gives us further insights in the mechanisms at work. Figure 4 shows how all putty-putty economies fail to generate any overshooting, in fact, only the economy with competitive wages manages to move the labour share at all (and only immediately after the shock). Those that have wage bargaining are subject to the puzzle noted by Shimer (2005): most of the productivity increases go to wages of existing workers, not to new workers. Only the economy with competitive wage setting displays movements in employment of any significance, although employment peaks when the shock hits and then it returns to the steady-state level pretty rapidly unlike both the data and the putty-clay economies. The competitive wage economy is an extreme form of what was proposed by Hagedorn and Manovskii (2008), in that the gap between productivity and wages (which is zero in the competitive wage case) is negatively related to the ability of the model to generate movements in employment.

A valuable lesson of looking at the Cobb-Douglas putty-putty economies is the extreme similarity in the business cycle performances of the dynamic and the static Nash bargaining economies (only the steady-state value of leisure changes to maintain average labour share). This gives peace of mind with respect to the assumption of static bargaining in the putty-clay economy: it seems that it is not extremely important to characterize the behaviour of non competitive wage setting environments.

Figure 4: Cobb-Douglas putty-putty Economies with Search and Matching with different wage settings

Impulse response functions



S&M-Dyn NB refers to a model with standard Nash Bargaining. S&M-Stat NB refers to a model with static Nash Bargaining. S&M-Comp W refers to a model with competitive wage setting.

Another piece of intuition that we gain when comparing putty-clay and Cobb-Douglas putty-putty economies, is that in the former, each additional hire does not decrease the productivity of all other workers in the economy as is the case in the latter. Thus, the incentives to hire workers in the model with putty-clay last longer.

Overall, the evidence points to competitive wages and a high degree of time bias as the model economy that best matches the data. It does so through its ability to generate a strong and relatively early response of employment without needing to generate extreme movements in the consumption to output ratio. It also benefits from the fact that the labour share responds more strongly to the product

of employment and the consumption ratio. Another feature that goes in its favour is that the need for an extreme time biased shock is not as strong as in the Nash bargaining case. The reason is that competition breaks the link between productivity in the plant and the salary of its worker which makes wages in new plants go up much less than in the Nash bargaining economies. Lower wages in new plants imply that a lower productivity is needed to make job creation attractive.

## 4.2 Estimating the Shock on Employment and the Consumption to Output Ratio

We want to know whether we can improve the performance of our putty-clay economies in terms of the responses of both employment and the consumption to output ratio, instead of focusing on the labour share alone. To this end, below we present an alternative estimation strategy where we target jointly the impulse responses of both the consumption-output ratio ( $C/Y$ ) and employment ( $N$ ) instead of the impulse response of the labour share.

Table 5: Targeting Employment and the Consumption Output Ratio: Estimated Processes

	Nash Bargaining	Competitive Setting
$\lambda$	0.0471 (0.015)	0.0504 (0.011)
$\rho$	0.979 (0.001)	0.983 (0.001)
$\sigma_\epsilon$	0.100 (0.024)	0.094 (0.015)
Sum Square Residuals of Labour Share IRF	2.24E-04	2.13E-04

Estimates when impulse responses of  $C/Y$  and  $N$  are targeted. Standard errors in parenthesis The last row displays the sum of square of the residuals between the impulse response function of labour share of the data and each model. Monthly process.

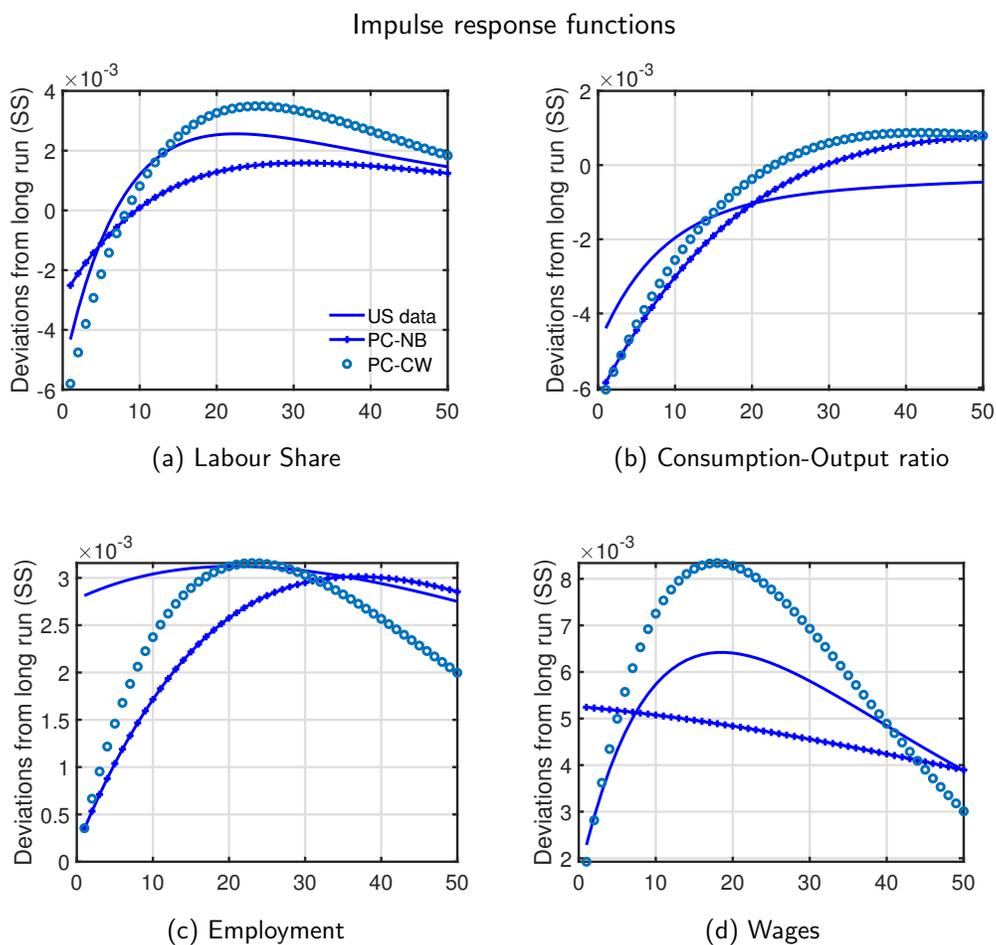
The estimated parameters are in Table 5 for both time biased economies while Figure 5 displays the associated impulse responses. The table shows that the parameter estimates are quite similar for both economies and that their ability to match the responses of the labour share is worse than the previous models, as shown by the sum of squared residuals.

The main difference with the results from the previous models ( Figures 1 and 3) is that the new estimation strategy yields bigger responses of both employment and the consumption to output ratio for

both economies, with the Nash bargaining economy producing less overshooting, and the competitive wage economy producing more.

As noted earlier, our model does not have a participation margin for workers: there are only employed and unemployed individuals. Thus, although putty-clay can yield a strong response from employment, the model is not yet capable of replicating the strong and speedy response observed in US data. Adding a participation margin in the model could attract individuals to work in expansions making it easier for firms to hire workers by increasing their availability. Such extension would also require a more explicit search model and we leave this for the future.

Figure 5: Putty-Clay Economies with Technical Time Bias (Alternative Estimation)



### 4.3 Other Business Cycle Statistics in Data and Model Economies

How do these model economies behave in terms of standard aggregate variables? Table 6 reports relative standard deviations (columns  $\sigma_x/\sigma_y$ ), correlations with output (columns  $\rho_{x,y}$ ) and autocorrelations (columns  $\rho_{x,x'}$ ). For model simulated data, we aggregate monthly series to quarterly data by simple averaging. For all series (data and simulated), we take logs and detrend using the Hodrick-Prescott filter, with smoothing parameter set to 1600, as is standard in the literature.

Table 6: Cyclical volatility of U.S. and Estimated Economies

<i>x</i>	US data			Nash Bargaining						Competitive					
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$	Time Bias			No time bias			Time Bias			No time bias		
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$												
<i>Emp</i>	0.64	0.80	0.90	0.44	0.49	0.96	0.20	0.33	0.96	0.20	0.43	0.96	0.18	0.42	0.96
<i>Une</i>	8.50	-0.87	0.89	10.27	-0.38	0.95	4.29	-0.28	0.96	3.86	-0.38	0.96	3.39	-0.38	0.96
<i>LSh</i>	0.66	-0.22	0.63	0.61	-0.64	0.78	0.26	-0.79	0.79	0.58	-0.73	0.78	0.53	-0.74	0.78
<i>Wag</i>	0.62	0.55	0.74	0.37	0.98	0.85	0.73	1.00	0.79	0.54	0.91	0.88	0.56	0.94	0.86
<i>Con</i>	0.74	0.78	0.81	0.66	-0.19	0.87	0.51	0.95	0.84	0.54	0.91	0.88	0.56	0.94	0.86
<i>Inv</i>	4.49	0.85	0.81	4.88	0.91	0.77	2.62	0.98	0.78	2.63	0.97	0.78	2.49	0.97	0.78

Note: US data from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:Q1-2018:QIV.  $\sigma_x/\sigma_y$  is the relative standard deviation of variable  $x$  with respect to output; while  $\rho_{x,y}$  is the correlation of variable  $x$  also with respect to output. All series are logged and HP-filtered.

The volatility of employment is more subdued than its data counterpart, pointing to the main shortcoming of these economies. The behaviour of other variables is much more in line with the data. Especially, that of unemployment, an artefact of the lack of a labour force participation margin. As hinted before, the time biased Nash bargaining economy has a very counterfactual property in terms of the correlation between consumption and output: it is negative, while this correlation is large and positive in the data (also across time periods and countries). We see that the small excessive response of wages to a technology innovation in the competitive wage setting economies does not translate into excessive wage moments. The persistence of all the variables are in line with the data.

Table 7: Cyclical volatility of Alternate Estimations with Time Bias

$x$	US data			Putty-clay - NB			Putty-clay - comp. w		
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$
<i>Employ</i>	0.638	0.803	0.902	0.257	0.378	0.962	0.255	0.468	0.961
<i>Unempl</i>	8.497	-0.872	0.893	6.339	-0.302	0.958	5.756	-0.388	0.960
<i>Labour Sh</i>	0.658	-0.216	0.633	0.339	-0.763	0.785	0.753	-0.699	0.781
<i>Wages</i>	0.625	0.553	0.742	0.642	0.999	0.791	0.472	0.748	0.938
<i>Cons</i>	0.743	0.780	0.808	0.409	0.795	0.913	0.472	0.748	0.938
<i>Invest</i>	4.483	0.848	0.806	3.139	0.970	0.777	3.107	0.952	0.775

Note: US data is taken from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:Q1-2018:Q4.  $y$  represents output;  $\sigma_x/\sigma_y$  is the relative standard deviation of variable  $x$  with respect to output;  $\rho_{x,y}$  is the correlation of variable  $x$  with respect to  $y$ . All series are logged and HP-filtered.

Table 8: Cyclical volatility of U.S., No time bias and Search and Matching models

$x$	US data			S&M dyn-NB			S&M stat-NB			S&M comp. w		
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	$\rho_{xx'}$									
<i>Employ</i>	0.64	0.80	0.90	0.08	0.94	0.83	0.11	0.94	0.84	0.62	0.97	0.82
<i>Unempl</i>	8.50	-0.87	0.89	1.30	-0.94	0.83	1.75	-0.94	0.84	12.25	-0.89	0.81
<i>Labour Sh</i>	0.66	-0.22	0.63	0.01	-0.92	0.69	0.01	-0.98	0.77	0.19	-0.77	0.52
<i>Wages</i>	0.62	0.55	0.74	0.92	1.00	0.79	0.88	1.00	0.78	0.28	0.86	0.87
<i>Cons</i>	0.74	0.78	0.81	0.33	0.90	0.85	0.26	0.89	0.85	0.28	0.86	0.87
<i>Invest</i>	4.48	0.85	0.81	3.19	0.99	0.79	3.69	0.99	0.80	3.27	0.99	0.83

Note: US data is taken from the Bureau of Labor Statistics (see Appendix C for details) for the period 1947:Q1-2018:Q4.  $y$  represents output;  $\sigma_x/\sigma_y$  is the relative standard deviation of variable  $x$  with respect to output;  $\rho_{x,y}$  is the correlation of variable  $x$  with respect to  $y$ . All series are logged and HP-filtered.

The standard business cycle statistics for the economies with search frictions and Cobb-Douglas putty-putty technology are in Table 8. As highlighted by Shimer (2005), this technology coupled with Nash bargaining with a high value of the workers bargaining weight has a hard time matching the aggregate volatility of employment and unemployment. The competitive wage setting can do a lot better, (as pointed to by Hagedorn and Manovskii (2008)), but, as noted by Choi and Ríos-Rull (2009), it does not generate the overshooting of the labour share. The main reason for this is the fact that employment peaks immediately after the shock hits and it does not have a sufficiently long-lived response to generate any overshooting.

Our main takeaway from this subsection is the ability of the putty-clay model with competitive wages to replicate volatility of labour market aggregates when faced with shocks estimated from data.

## 5 Conclusions

In this paper we have proposed a flexible way to introduce productivity shocks into a framework with putty-clay technology with an additional parameter whose value tells us whether we should think of technology shocks as being neutral or as being akin to investment specific shocks. The main insight from our analysis, is the fact that the latter can be thought of as neutral shocks which have a *time* bias, which favours recently created productive units. Our results show that, when targeting volatility of the Solow residual and the impulse response of the labour share in the US economy, a model with competitive wage setting and a strong time bias of the aggregate shock does best (quite well actually). In particular, it does better than a model with Nash bargaining and extreme time bias. Still, matching both the response of the consumption to output ratio and of employment is not yet achieved.

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## Appendix

### A Time Biased Shocks and Investment Specific Technological Innovations

To show the distinction between our time biased shock and standard investment specific technological (IST) shocks, in this section we depict the dynamics of both shocks. In Table 9 we show the time biased shocks, while the dynamics of standard investment specific shocks are in Table 10. For the latter, we imagine an economy where capital accumulation is given by  $K_{t+1} = (1 - \delta)K_t + e^{\tilde{z}_t} Q_t$ , where  $\tilde{z}$  evolves according to Equation (2) and  $Q_t$  is the amount of capital installed in period  $t$ .

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	$\lambda$	$\lambda$	$\lambda$	...
1	$\rho$	$\rho$	$\lambda\rho$	$\lambda\rho$	$\lambda\rho$	...
2	$\rho^2$	$\rho^2$	$\lambda\rho^2$	$\lambda\rho^2$	$\lambda\rho^2$	...
3	$\rho^2$	$\rho^3$	$\lambda\rho^3$	$\lambda\rho^3$	$\lambda\rho^3$	...
4	$\rho^3$	$\rho^4$	$\lambda\rho^4$	$\lambda\rho^4$	$\lambda\rho^4$	...
...	...	...	...	...	...	...

Table 9: Representation of shock dynamics.

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	0	0	0	0	...
1	$\rho$	1	0	0	0	...
2	$\rho^2$	$\rho$	1	0	0	...
3	$\rho^2$	$\rho^2$	$\rho$	1	0	...
4	$\rho^3$	$\rho^3$	$\rho^2$	$\rho$	1	...
...	...	...	...	...	...	...

Table 10: Representation of investment specific shocks.

For both tables, we imagine an economy in steady state, and introduce a shock to productivity of size one (just for exposition). Given the AR(1) representation of the shocks, in both economies the value of the shock declines according to  $\rho$ .

In our time-biased representation of the shock (see Equation (1) and Equation (2)), a machine that is 1 period old (installed just last period), enjoys productivity  $e^1$  the first period that is operational (time 0 in the table), and enjoys the declining productivity sequence  $\{\lambda\rho, \lambda\rho^2, \lambda\rho^3, \dots\}$ . Furthermore, all older machines enjoy the productivity shock, in amount  $\lambda$ , and as time passes, they also are exposed to a declining value of the shock.

On the other hand, in Table 10 we see that machines ( $Q$ ) which are 1 period of age, enjoy IST for all of their existence (sequence of 1's in the table). In a model with IST, installed machines do not enjoy these shocks (thus the zeros in the table).

		machine age (in periods)				
time	shock	1	2	3	4	...
0	1	1	0	0	0	...
1	$\rho$	$\rho$	0	0	0	...
2	$\rho^2$	$\rho^2$	0	0	0	...
3	$\rho^2$	$\rho^3$	0	0	0	...
4	$\rho^3$	$\rho^4$	0	0	0	...
...	...	...	...	...	...	...

Table 11: Representation of shock dynamics with  $\lambda = 0$

The extreme case of full time biased shocks, i.e.,  $\lambda = 0$ , is shown in Table 11. There, we can see the similarities and differences between this type of shock and standard IST. On the one hand, both shocks give the same value of the shock to the newest machines. However, the  $\lambda = 0$  case does not affect machines of any cohort beyond those 1 period old, which is the main difference with how IST affects dynamically investment. However, although our formulation removes the effect of the shock on older cohorts of machines/capital, these are depreciating at exponential rate  $\delta$ , giving more weight to the newest cohorts.

Thus, although there are strong similarities between IST and our shock when  $\lambda = 0$ , they are not the same.

## B Standard Search and Matching Model in General Equilibrium

We use the model put forward by Andolfatto (1996), which is a general equilibrium version of the standard search and matching framework. The model is described by perfect insurance inside the household (so employed and unemployed workers consume the same) and search and matching frictions in the labour market. Households derive utility from consumption and the leisure enjoyed by its members, with per period utility given by  $\log(C) + (1 - N)b$ , where  $C$  is consumption,  $N$  is the measure of household

members who are unemployed while  $b$  is a parameter related to the extra amount of leisure enjoyed by the unemployed. Households decide how much to consume and save every period. Firms produce using a standard Cobb-Douglas production function  $Y = e^z K^\alpha N^{1-\alpha}$ , rent capital  $K$  and search for workers by posting vacancies  $V$ , at flow cost  $c^v$ . Matches in the economy are given by the matching function  $m(V, 1 - N) = \omega V^\psi (1 - N)^{1-\psi}$  and there is a lag of one period between match and production. Real wages are the solution to bilateral Nash bargaining between the firm and the household, with the bargaining weight of the worker being  $\mu$ . We consider two alternative versions of the model: (i) dynamic Nash bargaining and (ii) static Nash bargaining (to mimic our baseline economy).

The following equations characterize the equilibrium of this model (where primes denote future values):

Aggregate consistency and evolution of states:

$$Y = e^z K^\alpha N^{1-\alpha}$$

$$Y = I + C + c^v V$$

$$N' = (1 - \chi)N + \omega V^\psi (1 - N)^{1-\psi}$$

$$K' = (1 - \delta)K + I$$

$$z' = \rho z + \epsilon, \quad \epsilon \sim N(0, \sigma_\epsilon^2)$$

Job Filling Rate:

$$\Phi = \omega V^{\psi-1} (1 - N)^{1-\psi}$$

Euler equation:

$$1 = \beta E \left[ \frac{C}{C'} \left( 1 - \delta + (1 - \alpha) \frac{Y'}{K'} \right) \right]$$

Optimal vacancy posting decision:

$$\frac{c^v}{\Phi} = \beta E \left[ \frac{C}{C'} \left( \alpha \frac{Y'}{N'} - w' + (1 - \chi) \frac{c^v}{\Phi'} \right) \right]$$

Wages:

$$w = \mu \left[ \alpha \frac{Y}{N} + c^v \frac{V}{1 - N} \right] + (1 - \mu) C b \quad (\text{Dynamic bargaining}),$$

$$w = \mu \left[ \alpha \frac{Y}{N} \right] + (1 - \mu) C b \quad (\text{Static bargaining}).$$

## C Data and Estimation

Data is taken from the Bureau of Labor Statistics (BLS)'s website, the FRED site of the Saint Louis Fed and the Bureau of Economic Analysis (BEA) tables. The series we use are Output (Table 1.1.6., Real Gross Domestic Product in Chained Dollars from BEA), Labour Share Index (BLS series PRS85006173),

Employment (BLS series LNS12000000Q), Unemployment (BLS series LNS13000000), Real Hourly Compensation (BLS series PRS85006153), Personal consumption expenditures (FRED series PCECC96), Real Gross Private Domestic Investment (FRED series GPDIC96) and the chain-type quantity Indexes for Net Stock of Fixed Assets and Consumer Durable Goods, table 1.2 from the Fixed Asset tables from the BEA (as capital). All variables refer to non-farm business sector and are seasonally adjusted. We further normalize all variables by the non-institutionalized population in the US (BLS series LNU00000000Q). For all series, we consider information between the first quarter of 1948 and the fourth quarter of 2018 (last available data point for fixed assets).

As for the Solow residual, we follow closely the procedure in Ríos-Rull and Santaeulalia-Llopis (2010). It is defined as

$$\widehat{z}_t = y_t - \overline{LSH} n_t + (1 - \overline{LSH}) k_t,$$

where  $\overline{LSH}$  is the average labour share,  $y_t$  is detrended output,  $n_t$  is detrended employment and  $k_t$  is detrended capital, where the detrending procedure is linear: For variable  $X$ , it's residual is

$$x_t = \log(X_t) - \alpha_t^x t$$

where  $t$  is a linear trend and  $\alpha_t^x$  is the least square estimator of a linear trend on an equation on  $\log(X_t)$ .

From here, a regression of the form

$$\widehat{z}_t = \gamma \widehat{z}_{t-1} + \eta_t$$

produces a linear least squares estimate for the persistence,  $\hat{\gamma}$  and the volatility of the Solow residual,  $\hat{\sigma}_\eta$ .

In our estimation exercise (and across all models we present) we estimate the same equation using simulated data and match the estimates from our data sources:

$$\hat{\gamma} = 0.9611774 \quad (0.0137514)$$

$$\hat{\sigma}_\eta = 0.0077764 \quad (0.00032759)$$

where number in parentheses are standard deviations from the estimates. To maintain comparability across models in terms of the overall amount of volatility that they are able to generate, we force each model to match  $\hat{\gamma}$  and  $\hat{\sigma}_\eta$  almost perfectly. We accomplish this by way of increasing the weight of those targets in the estimation routine, when minimizing the sum of squared residuals between data and model generated moments. In total, we target 32 moments: the two moments related to the Solow process plus 30 elements in the impulse response function of labour share. We give the first two targets a weight of one thousand while the rest receive a weight of one. In all our estimations, the models match the estimates of  $\hat{\gamma}$  and  $\hat{\sigma}_\eta$  very accurately.

## D Labour Share During the Business Cycle

Here we look in detail at the behaviour of the labour share. On the left panel of Figure 6 we depict the labour share series (described in the previous section). We can see clearly its downward trend, which becomes even more pronounced around the mid 2000s. On the right panel, we present the estimated orthogonal impulse response from a vector autoregression of order 1 between linearly detrended log labour share and linearly detrended log GDP with its corresponding 95% confidence intervals.<sup>23</sup>

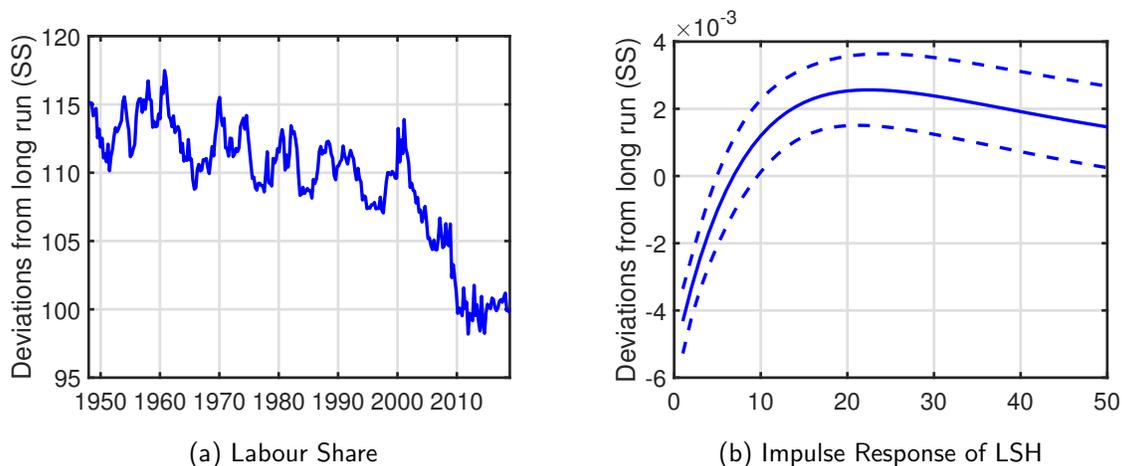


Figure 6: Left panel is a labour share Index for non-farm business US, 1948:Q1 to 2018:QIV. Right panel is an impulse response function of labour share to a shock in output, from a VAR(1) model. Source: Bureau of Labor Statistics.

The figure summarizes various facts with respect to the labour share which have been previously discussed in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaaulalia-Llopis (2010): Upon impact, labour share falls giving rise to the well known counter cyclical behaviour of the series. In time, labour share starts to rise, and peaks around 15 to 20 quarters after the initial shock. This positive response is persistent and long lasting, making the aggregate gains of labour due to an expansion positive in the medium to long run, more than compensating the initial drop in this share (hence the overshooting label). The estimates of the bivariate VAR show no effects whatsoever of lagged labour share on productivity. See Ríos-Rull and Santaaulalia-Llopis (2010) for details. Also, given that we are using a different time period and different variable definitions, this is also a sign of the robustness of the overshooting fact. As seen in the figures, this response is statistically significant at the 95% level.

When we refer to empirical impulse responses to other aggregates (consumption to output ratio, employment, average wages for example) in the main text, the procedure we follow to compute them, both in the model and US data, is analogous to what we have described above for the response of the labour share.

<sup>23</sup>See Hamilton (1994).

## Labour Share Trends

We turn to explore whether our findings are an artefact of the detrending. As seen in the left panel of Figure 6, the labour share exhibits a clear trend downwards, perhaps becoming steeper after the turn of the century. So we perform our analysis also on a shorter sample that avoids the recent years of falling labour share and by comparing the extent to which detrending affects the impulse response.

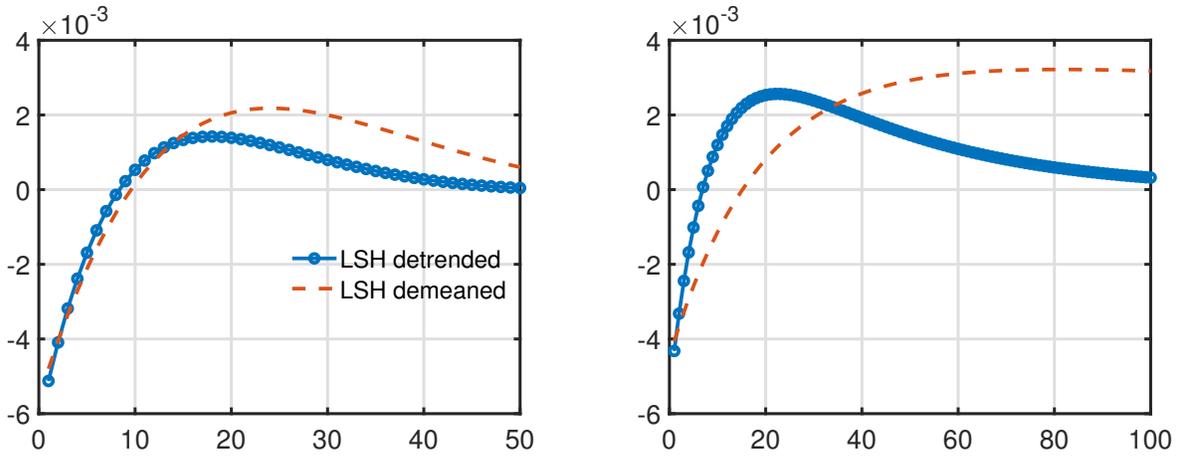


Figure 7: Impulse response functions of labour share in US data. Left panel: sample used in Ríos-Rull and Santaaulalia-Llopis (2010) (1954:Q1 to 2004:Q4). Right panel: full sample (1948:Q1 to 2018:Q4).

Figure 7 depicts the impulse response functions of the labour share as described above, both log detrended and not (just demeaned). In the left panel we use the same sample used when the overshooting property was first noted by Ríos-Rull and Santaaulalia-Llopis (2010) which spans 1954:Q1 to 2004:Q4, while the right panel contains our full sample. As seen from the figure, overshooting is present irrespective of whether we detrend or not. Moreover, the procedure of linearly detrending the labour share we use throughout the exercise in the main text, produces a lower overshooting versus when we do not. Further, we see that detrending pulls the overshooting a bit earlier.

We estimate the shock process in the model with these alternative impulse responses and the results are depicted in Tables 12 and 13. As seen from both tables, the main message of the main exercise is maintained, in that in both subsamples and detrending procedures, the estimation finds low values of  $\lambda$ , implying high levels of time bias. On the other hand, the sum of squared residuals of the labour share IRF is not as clear in preferring competitive wage setting as in the main text. However, the competitive wage setting still behaves better in terms of the relative performance of the consumption aggregate. As noted in the main exercise in the text, the economy with Nash bargaining and time bias has a counterfactual behaviour in terms of consumption: it is counter-cyclical with respect to output. In Table 14 and Table 15 we see that, even though this countercyclical feature does not repeat in all cases, all competitive wage economies are closer to the data in terms of the correlation of consumption relative to output ( $\rho_{x,y}$ ) and, in all but the estimation on the “full sample, detrended labour share”,

Table 12: Model Estimated Using 1954-2004 Sample

	Nash Bargaining		Competitive Setting	
	Detrended	Not	Detrended	Not
$\lambda$	0.075 (0.028)	0.015 (0.004)	0.491 (0.488)	0.289 (0.233)
$\rho$	0.982 (0.002)	0.982 (0.003)	0.987 (0.001)	0.986 (0.002)
$\sigma_\varepsilon$	0.066 (0.020)	0.237 (0.033)	0.011 (0.010)	0.018 (0.014)
Sum Square Residuals of Labour Share IRF	4.65E-05	2.06E-05	6.97E-05	1.46E-05

Table 13: Model Estimated Using Full (1948-2018) Sample

	Nash Bargaining		Competitive Setting	
	Detrended	Not	Detrended	Not
$\lambda$	0.005 (0.001)	0.015 (0.005)	0.218 (0.126)	0.2252 (0.164)
$\rho$	0.973 (0.003)	0.977 (0.004)	0.984 (0.002)	0.984 (0.003)
$\sigma_\varepsilon$	0.408 (0.035)	0.234 (0.042)	0.024 (0.013)	0.024 (0.016)
Sum Square Residuals of Labour Share IRF	1.58E05	4.11E-05	4.35E06	7.44E-05

the competitive wage setting produces relative volatilities of consumption more in line with the data. However, this latter case is where the Nash bargaining economy with time bias produces counter-cyclical consumption.

Table 14: Cyclical volatility of U.S. and Estimated Economies (1954-2004 sample)

<i>x</i>	US data		Detrended				Not detrended			
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	Nash bargaining		Competitive		Nash bargaining		Competitive	
	$\sigma_x/\sigma_y$	$\rho_{x,y}$								
<i>Employ</i>	0.62	0.80	0.23	0.36	0.18	0.42	0.33	0.42	0.18	0.42
<i>Unempl</i>	7.31	-0.87	5.53	-0.29	3.38	-0.38	8.06	-0.31	3.56	-0.38
<i>Labour Sh</i>	0.69	-0.30	0.29	-0.78	0.52	-0.74	0.44	-0.72	0.53	-0.74
<i>Wages</i>	0.60	0.49	0.69	1.00	0.58	0.95	0.53	1.00	0.56	0.94
<i>Cons</i>	0.81	0.88	0.46	0.90	0.58	0.95	0.41	0.36	0.56	0.94
<i>Invest</i>	4.19	0.91	2.85	0.98	2.44	0.97	3.78	0.95	2.49	0.97

Note:  $\sigma_x/\sigma_y$  is the relative standard deviation of variable *x* with respect to output; while  $\rho_{x,y}$  is the correlation of variable *x* also with respect to output. All series are logged and HP-filtered.

Table 15: Cyclical volatility of U.S. and Estimated Economies (full 1948-2018 sample)

<i>x</i>	US data		Detrended				Not detrended			
	$\sigma_x/\sigma_y$	$\rho_{x,y}$	Nash bargaining		Competitive		Nash bargaining		Competitive	
	$\sigma_x/\sigma_y$	$\rho_{x,y}$								
<i>Employ</i>	0.64	0.80	0.44	0.49	0.20	0.43	0.34	0.43	0.20	0.43
<i>Unempl</i>	8.50	-0.87	10.27	-0.38	3.86	-0.38	8.25	-0.33	3.84	-0.38
<i>Labour Sh</i>	0.66	-0.22	0.61	-0.65	0.58	-0.73	0.46	-0.71	0.58	-0.73
<i>Wages</i>	0.62	0.55	0.37	0.97	0.54	0.91	0.52	1.00	0.54	0.91
<i>Cons</i>	0.74	0.78	0.66	-0.19	0.54	0.91	0.42	0.28	0.54	0.91
<i>Invest</i>	4.48	0.85	4.88	0.91	2.63	0.97	3.89	0.95	2.62	0.97

Note:  $\sigma_x/\sigma_y$  is the relative standard deviation of variable *x* with respect to output; while  $\rho_{x,y}$  is the correlation of variable *x* also with respect to output. All series are logged and HP-filtered.

### Robustness of overshooting: Long run restriction

On the left panel of Figure 8, we show impulse response functions of labour share: the solid line is ours, while the dotted line represents the impulse response function from a SVAR with long run restrictions. We follow the baseline estimation in Santaaulalia-Llopis (2012), who uses a standard procedure to identify technology shocks: that the effect of technology on labour inputs is zero in the long run, as is done in Galí (1999b) and Fisher (2006b). We can see that the overshooting fact is maintained under the standard SVAR specification, even if slightly smaller.

### Robustness of overshooting: Fernald's adjusted TFP

We also look at whether the overshooting property of the labour share occurs when using Fernald's utilization adjusted TFP series.<sup>24</sup> The right panel of Figure 8 shows the impulse response of labour share. The overshooting shape is there, but the size is not. We do not see this as evidence against our analysis. The Fernald TFP series is an attempt to get at some true technology series that takes

<sup>24</sup>Downloaded from

<https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp>. See Fernald (2012).

into account some variability in capital utilization. As such it is not that different than what our own model could produce: a technology series conditional on a particular theory. Our simple model does not contain a mechanism to vary factor utilization so it cannot speak directly to what produces this lower reaction of labour share to this alternative TFP measure.

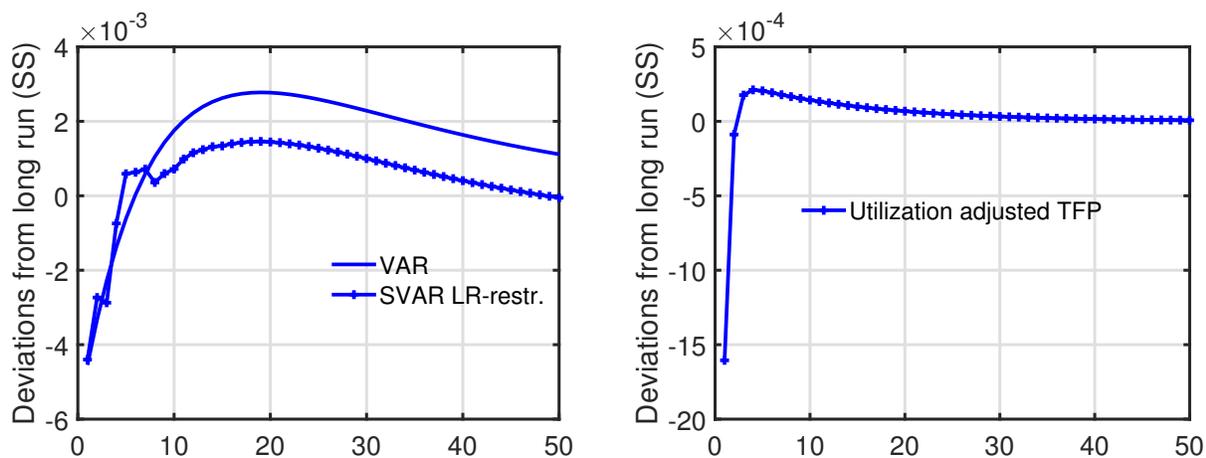


Figure 8: Impulse response functions of labour share in US data: structural vector autoregression with long-run restrictions (left) and VAR using Fernald's utilization TFP series. (right).