Online Appendix

Revisiting the Effect of Household Size on Consumption Over the Life-Cycle

Not intended for publication

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July 2013

Abstract

This appendix contains four sections supplementing the analysis in the main paper. First, we provide a further theoretical result which discusses differences in the level of per-adult equivalent consumption in the *Demographics* and *Single Agent* model. Second, we show and discuss insurance coefficients in the spirit of Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010) for our benchmark calibration. Third, we report the quantitative results for our benchmark model extended with a pensions system. Fourth, based on the empirical estimates in Attanasio, Banks, Meghir, and Weber (1999) we back out the implied utility weights δ in the *Demographics* model.

1 Consumption Levels

Figure 1a in the paper compares the *Single Agent* model with the *Demographics* model with $\delta = \phi$. One can directly see that the (life-time) level of per-adult equivalent consumption differs between the two models. The following section provides the theoretical explanation for this.

Proposition 1. *Life-time per-adult equivalent consumption in the* Demographics *model and* Single Agent *model coincide if and only if* $\kappa_2 = \frac{\phi_2}{1-(\phi_2-1)\frac{\mathbb{Y}_1}{\mathbb{Y}_2}\left(1-\frac{\mathbb{C}_1^D}{\mathbb{Y}_1}\right)}$.

This result is straightforward to show. Life-time per-adult equivalent consumption in the *Demographics* model $\mathfrak{c}_D$ is given by

$$\mathfrak{c}_D = \mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{\phi_2} = \mathbb{C}_1^D + \frac{\overbrace{\mathbb{Y}_1 + \mathbb{Y}_2 - \mathbb{C}_1^D}^{=\mathbb{C}_2^D}}{\phi_2} = \frac{\phi_2 - 1}{\phi_2}\mathbb{C}_1^D + \frac{\mathbb{Y}_1 + \mathbb{Y}_2}{\phi_2} \tag{1}$$

while in the *Single Agent* model life-time per-adult equivalent consumption $\mathfrak{c}_S$ equals life-time per-adult equivalent income:

$$\mathfrak{c}_S = c_1^S + c_2^S = \mathbb{Y}_1 + \frac{\mathbb{Y}_2}{\kappa_2}. \tag{2}$$

Equating Equations (1) and (2) yields the critical value of κ_2 stated in Proposition 1 which also has a very intuitive interpretation. One can directly see from Equations (1) and (2) that when the household in the *Demographics* model optimally chooses to neither save nor borrow ($\mathbb{C}_1^D = \mathbb{Y}_1$), the life-time per-adult equivalent consumption levels in both setups are the same if and only if $\kappa_2 = \phi_2$. Now consider the case when the household in the *Demographics* model optimally chooses to be a borrower ($\mathbb{C}_1^D > \mathbb{Y}_1$). Assume for a moment that κ_2 is still equal to ϕ_2 instead of the critical value stated in Proposition 1. In the *Demographics* model only period two consumption — which in this example is smaller than period two income — is deflated by the equivalence scale in the calculation of life-time per-adult equivalent consumption. This directly implies a higher life-time per-adult equivalent consumption level in the *Demographics* model compared to the *Single Agent* model which is determined only by life-time per-adult equivalent income. To undo this, κ_2 has to be decreased below ϕ_2 in order to increase life-time per-adult equivalent income and thus life-time per-adult equivalent consumption in the *Single Agent* model.

While in quantitative work life-time (per-adult equivalent) consumption is not a statistic of primary interest, it is relevant for welfare comparisons between different economic environments. If e.g. a linear income tax rate is raised to finance wasteful government consumption, households in the economy with the lower, pre-reform life-time per-adult equivalent consumption suffer a larger welfare loss with concave utility. Moreover, this statistic highlights a key difference between the *Single Agent* and *Demographics* model. In the latter the household has total household income at its disposal to allocate between the two periods which then determines life-time per-adult equivalent consumption. In the *Single Agent* model, the income process is exogenously adjusted thus directly restricting life-time per-adult equivalent income (total resources available). This difference becomes particularly obvious when one would consider two types of households (A, B) experiencing differences in the timing of household income, i.e. $\frac{y_1^A}{y_2^A} \neq \frac{y_1^B}{y_2^B}$. Even for the same life-time household income $y_1^A + y_2^A = y_1^B + y_2^B$, life-time per-adult equivalent incomes differ in the *Single Agent* but not in the *Demographics* model. This generates inequality in life-time per-adult equivalent consumption in the *Single Agent* model but not in the *Demographics* model.

The derivation and implications of Proposition 1 only depend on the choice of δ_2 in so far that it determines $\mathbb{C}_1^D$ and thus, for a given $\mathbb{Y}_1$ and $\mathbb{Y}_2$, the relationship between the two per-adult equivalent consumption levels.

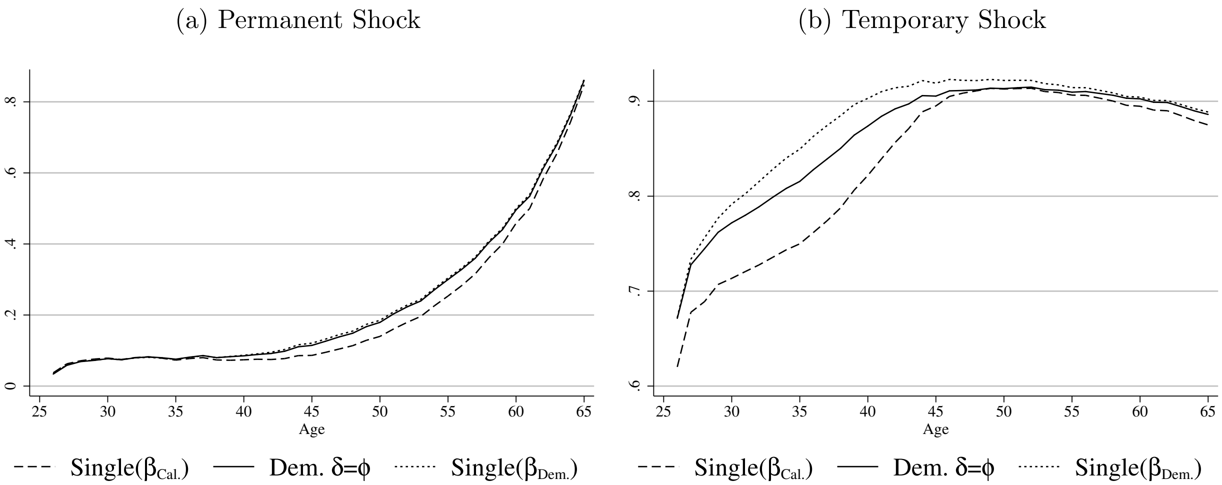
2 Insurance Coefficients

In the following section, we consult an alternative statistic closely related to the variance of log per-adult equivalent consumption. We follow Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010) and compute insurance coefficients for each model, a measure of how much consumption comoves with income shocks. For each model we calculate the contemporaneous correlation of changes in consumption and permanent (P) and transitory (Tr) shocks to labor income:

$$\psi_t^x = 1 - \frac{cov(\Delta \log(c_t/\phi_t), \epsilon_t^x)}{var(\epsilon_t^x)}$$

where c_t/ϕ_t is per-adult equivalent consumption at age t and $x = \{P, Tr\}$. The higher the comovement between consumption and unexpected changes in income (represented by the shocks ϵ^x), the lower is the implied value of the insurance coefficient.

Figure 1: Insurance Coefficients, *Single Agent* vs. *Demographics* with $\delta = \phi$



2.1 *Demographics* model with $\delta = \phi$ vs. *Single Agent* model

Figure 1a shows the insurance coefficient for the permanent income shock for the *Demographics* model with $\delta = \phi$ and the two calibrations of the *Single Agent* model (one with the discount factor being calibrated and one with the same discount factor as in the *Demographics* model). The life-cycle profiles of the insurance coefficients for the permanent shock mirrors the one for the log variance of per-adult equivalent consumption (Figure 1b in the paper). They are hardly distinguishable across the three simulations until the mid 30s, and diverge subsequently. The *Single Agent* model with the calibrated β displays a higher variance of log per-adult equivalent consumption, i.e. a higher consumption inequality, and the insurance coefficient is lower, i.e. per-adult equivalent consumption fluctuates more with the permanent income shock. The overall shape of these profiles is in line with the findings by Kaplan and Violante (2010). As households age and accumulate more assets they can better insulate their consumption from income shocks. Moreover, as the retirement age gets closer, the permanent shocks 'loose' in some sense their permanent nature, e.g. in the last period of working life the permanent shock is effectively a transitory shock.

Figure 1b compares the insurance coefficients for the transitory income shocks. Per-adult equivalent consumption comoves to a much lesser degree with transitory income shocks than with permanent income shocks. At age 26 the coefficients are between 0.6 and 0.7 for the transitory shock compared to less than 0.1 for the permanent shock. From the mid 40s onwards the insurance coefficients are very similar across the three models. When holding fixed the discount factor, households in the *Demographics* model are less insulated against transitory income shocks than in the *Single Agent* model because of the lower effective return to savings and thus higher costs of providing insurance. The *Single Agent* model calibrated to match the empirical wealth to income ratio has a lower discount factor. This higher preference for current consumption makes per-adult equivalent consumption by construction react more to income shocks (higher comovement, lower coefficient). Finally, the overall pattern looks closer to the case with a zero borrowing constraint in Kaplan and Violante (2010). The lack of a redistributive pension system in our benchmark analysis induces households to accumulate savings right from the start of their life rather than to smooth transitory shocks by going into debt.

To sum up, the permanent insurance coefficients are basically replicating the informational content of the variance of log per-adult equivalent consumption shown in the paper. The differences between the insurance coefficients for the transitory income shocks until the mid 40s are much larger than for any statistic we have reported. Since households can self-insure these types of shocks relatively well, these differences do not change our conclusion that the *Single Agent* model does not miss much relative to the *Demographics* model.

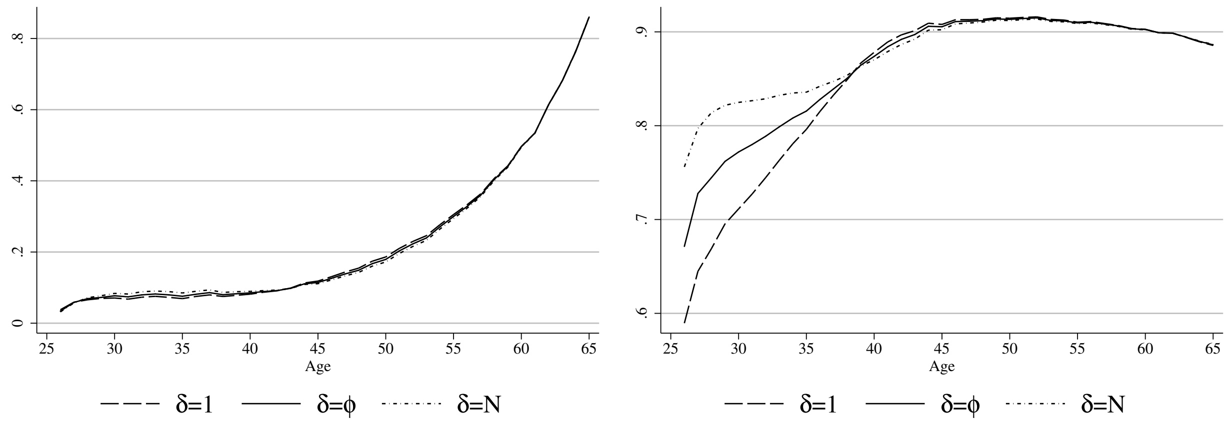
2.2 Different Specifications of the *Demographics* Model

Figure 2a shows the insurance coefficient for the permanent income shock for the three specifications of the *Demographics* model. Similar to the variance of log per-adult equivalent consumption consumption, the insurance coefficients are virtually the same.

Figure 2b compares the insurance coefficients for the transitory income shocks. From age 40 onwards the insurance coefficients are very similar across the three models. During the 20s and 30s, the higher preference for current consumption for the $\delta = 1$ specification makes consumption move more than for the $\delta = N$ speficiation. These differences, albeit being larger than in the previous case, remain of second order magnitude.

Figure 2: Insurance Coefficients, Different Specifications of the *Demographics* Model

(a) Permanent Shock (b) Temporary Shock



3 Introducing Pensions

Our benchmark calibration abstracts from pensions. Here we show the results with the same redistributive pension system as in Storesletten, Telmer, and Yaron (2004). Pensions are fully funded by a proportional tax on labor income. Pensions themselves depend on average income over the life-cycle as follows:

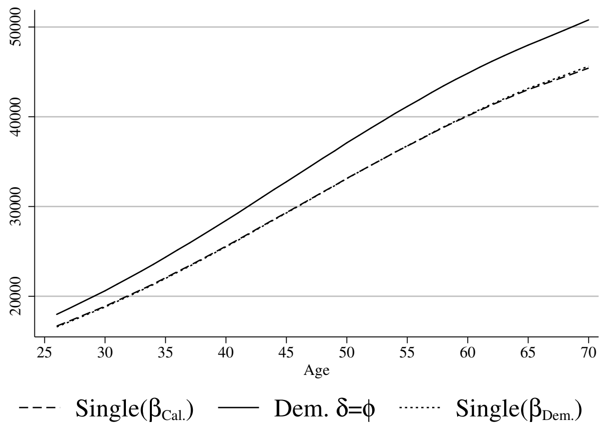
$$p(\bar{y}_i) = \begin{cases} \quad 0.90 \times \;\min\{\bar{y}_i, 0.3\bar{y}\} & \text{if } \bar{y}_i \leq 4.1\bar{y} \\ + \quad 0.32 \times (\min\{\bar{y}_i, 2.0\bar{y}\} - 0.3\bar{y}) & \text{if } 0.3\bar{y} < \bar{y}_i \leq 4.1\bar{y} \\ + \quad 0.15 \times (\min\{\bar{y}_i, 4.1\bar{y}\} - 2.0\bar{y}) & \text{if } 2.0\bar{y} < \bar{y}_i \leq 4.1\bar{y} \\ \quad 1.10\bar{y} & \text{if } \bar{y}_i > 4.1\bar{y}, \end{cases}$$

where $\bar{y}_i$ is the individual average of life-time earnings and $\bar{y}$ is the average of cross-sectional earnings in the economy. Households with an average life-time income more than 4.1 times the average cross-sectional earnings receive a pension of 1.1 times the average cross-sectional earnings. Household with average life-time earnings below this threshold receive a replacement rate of 90% for any fraction of their life-time earnings that falls below 30% of average cross-sectional earnings. Any fraction of life-time earnings above this threshold and below twice average cross-sectional earnings is replaced at a rate of 32% and any fraction of life-time earnings between 2 and 4.1 times the average cross-sectional earnings is replaced at a rate of 15%.

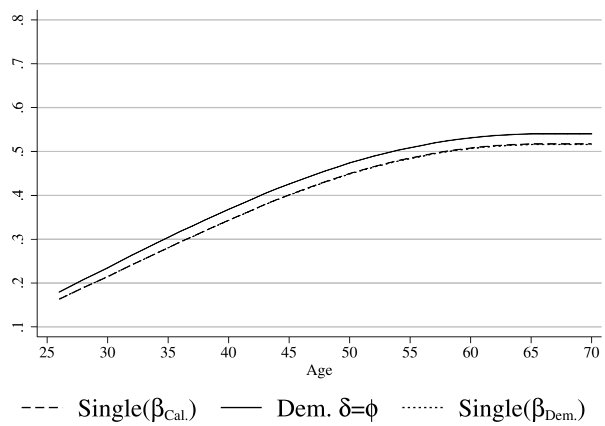
Figure 3: *Single Agent* vs. *Demographics* with $\delta = \phi$ and US style pension system

Per-Adult Equivalent Consumption

(a) Mean

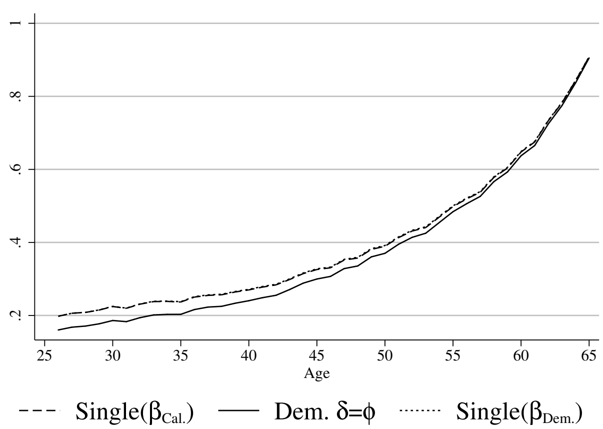


(b) Variance (of Log)

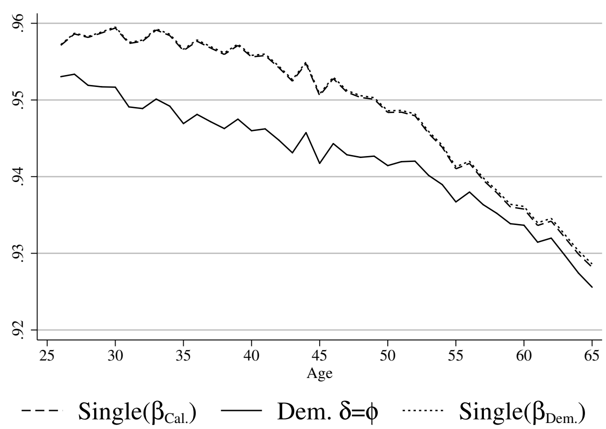


Insurance Coefficients

(c) Permanent Shock



(d) Temporary Shock



During working life households in the *Demographics* model receive household income whereas in the *Single Agent* model households receive household income divided by the adjustment factor κ, i.e. per-adult equivalent income. We maintain this analogy during retirement. A household in the *Single Agent* model who experiences the same sequence of permanent and transitory income shocks as a corresponding household in the *Demographics* model is assigned the same pension as the household in *Demographics* model, but divided by the adjustment factor κ.

We leave all preference parameters and prices as in our benchmark simulation without a pension system. But as before, we calibrate for each simulation the discount factor β in order to generate a wealth to income ratio of 3.1. Since the introduction of a redistributive pension system reduces the savings motive for retirement, all discount factors increase relative to the respective setting without a pension system.

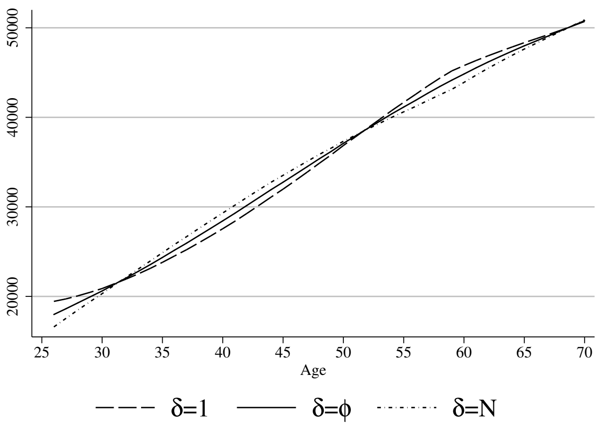
3.1 *Demographics* model with $\delta = \phi$ vs. *Single Agent* model

The calibrated βs for the *Demographics* model with $\delta = \phi$ and the *Single Agent* model are 1.0 and 0.999, respectively, which generate upward sloping consumption profiles even during retirement, see Figure 3a. Consumption inequality is reduced substantially compared to the no pension economies, specifically during retirement, see Figure 3b. Since the calibrated discount factors are virtually identical in the *Demographics* and *Single Agent* model, the specification of the *Single Agent* model with the same discount factor as in the *Demographics* model produces virtually the same results as the *Single Agent* model with the calibrated discount factor. Put differently, in contrast to the scenario without pensions, only the difference in the effective interest rate matters. Thus, because of the lower effective interest rate and higher cost of providing insurance the variance of log per-adult equivalent consumption is now larger in the *Demographics* model over the entire life-cycle with a difference of 4% during retirement. This is also reflected in lower insurance coefficients for both shocks, although the differences are fairly small, see Figures 3c and 3d. It is worthwhile to mention that with the pension system in place, the insurance coefficients look more similar to those in Kaplan and Violante (2010) with the natural borrowing constraint. Households now do in fact borrow during the early part of life and thus are better insured against income shocks, specifically transitory ones.

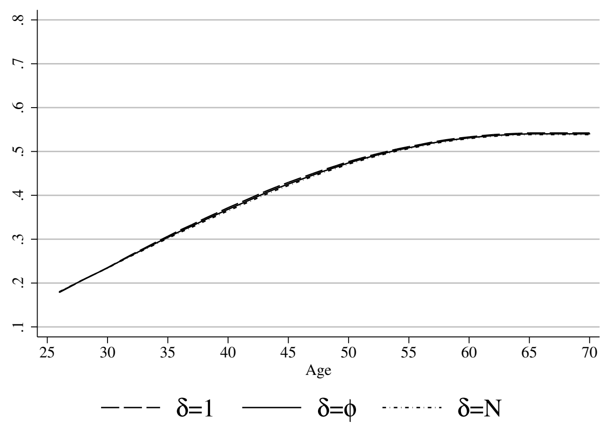
Figure 4: Different Specifications of the *Demographics* Model with US style pension system

Per-Adult Equivalent Consumption

(a) Mean

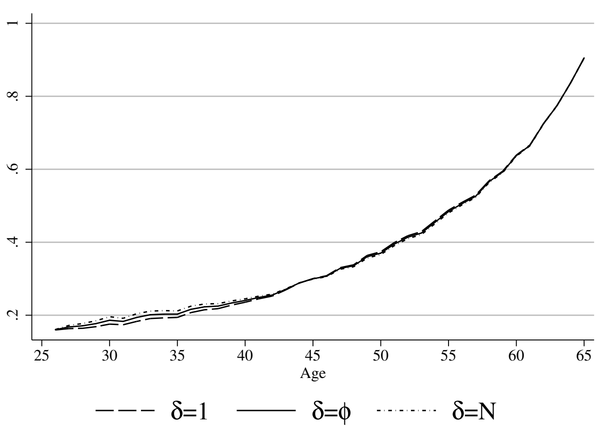


(b) Variance (of Log)

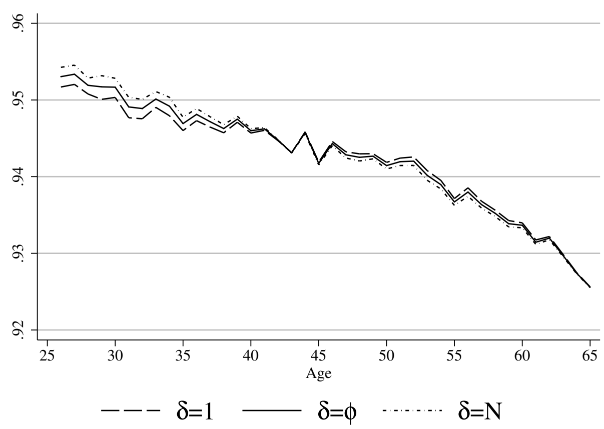


Insurance Coefficients

(c) Permanent Shock



(d) Temporary Shock



3.2 Different Specifications of the *Demographics* Model

The calibrated discount factors are $\beta = 0.992$ for $\delta = 1$, $\beta = 1.0$ for $\delta = \phi$ and $\beta = 1.01$ for $\delta = N$. The qualitative and quantitative differences across the three different choices for δ are similar to those obtained without a pension system, see Figures 4a and 4b. As in the previous subsection, the pattern for the insurance coefficients are similar to those in Kaplan and Violante (2010) with the natural borrowing constraint, see Figures 4c and 4d.

4 Some Empirical Evidence

As already mentioned, Attanasio, Banks, Meghir, and Weber (1999) (henceforth ABMW) introduce a general taste shifter to capture the impact of household size on the marginal utility of per-adult equivalent consumption

$$u(\mathbb{C}_t, N_t) = \exp(\xi N_t) u\left(\mathbb{C}_t\right). \tag{3}$$

while preferences in our model are

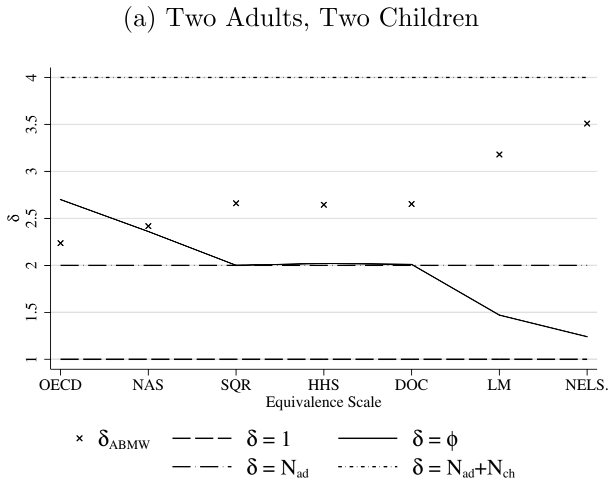
$$u(\mathbb{C}_t^D, N_t) = \delta(N_t) u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right). \tag{4}$$

Despite the fact that the choice of δ does not generate important differences for key model predictions - as long as being calibrated to match aggregate moments - ABMW's results provide some empirical guidance for the choice of the utility weight δ. With CRRA preferences and a given coefficient of relative risk aversion, their specification of the utility function coincides with the one used in our exercise if the following relationship holds:

$$\exp\left(\zeta_1[N_{ad} - 1] + \zeta_2 N_{ch}\right) = \frac{\delta(N_{ad}, N_{ch})}{\phi(N_{ad}, N_{ch})^{1-\alpha}}, \tag{5}$$

where we normalize the taste shifter from ABMW to one for households of size one. ABMW log-linearize the Euler equation implied by Equation (3), assuming that households can borrow up to the natural borrowing constraint, and estimate ζ_1, ζ_2, and α from CEX data. With those estimates ($\hat{\zeta}_1 = 0.71, \hat{\zeta}_2 = 0.34$ and $\alpha = 1.57$) at hand, one can back out the δ that solves Equation (5) for a given household size and composition, and a given equivalence scale ϕ (henceforth labeled as

Figure 5: Utility Weights for various Equivalence scales



δ_{ABMW}). We calculate the implied δ for each of the seven equivalence scales discussed in Fernández-Villaverde and Krueger (2007) and compare them to the set of δs used in applied work. We use the Square Root Scale instead of the 'Mean' scale in Fernández-Villaverde and Krueger (2007) as the two are almost identical.

Figure 5a refers to a household with two adults and two children. The horizontal axis displays the seven equivalences scales ordered from the lowest economies of scale (OECD) to the highest economies of scale (Nelson), whereas the vertical axes refers to the values the various δs take. The four lines refer to the set of δs used in applied work. All $\delta s \in \{1, N_{ad}, N = N_{ad} + N_{ch}\}$ are independent of the respective equivalence scale and therefore parallel to the horizontal axis. $\delta = \phi$, the solid line, obviously varies with the equivalence scale. Finally, the crosses refer to the δ_{ABMW} implied by the estimates in ABMW for a given equivalence scale. The figure has to be read as follows: for the OECD scale δ_{ABMW} is closest to $\delta = N_{ad}$, whereas for the NAS scale it is virtually identical to $\delta = \phi$. Overall, $\delta = \phi$ and $\delta = N_{ad}$ (number of adults in the household) most closely resemble the δ implied by the estimates in ABMW for the equivalence scales featuring low to medium economies of scale (OECD to DOC), whereas for the equivalence scales featuring high economies of scale (LM and Nelson) $\delta = N_{ad} + N_{ch}$ (household size) stands out.

Figure 5b shows the life-cycle averages for each possible δ using the household size and composition profiles from our quantitative analysis, i.e. $\frac{1}{71}\sum_{t=25}^{95}\delta(N_{ad,t}, N_{ch,t})$. Again, $\delta = N_{ad}$ is relatively close to all equivalence scales, whereas $\delta = \phi$ only for those scales featuring low to medium economies of scale. All this together suggest to not use the 'extreme' choices for δ, namely one and household size. However, this conclusion relies on taking the estimates by Attanasio, Banks, Meghir, and Weber (1999) for ξ_1, ξ_2 and the coefficient of risk aversion at face value.

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